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Dynamically Stable Legged Locomotion

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This report summarizes progress over the past 3 yrs. in exploring active balance for dynamic legged systems. The purpose of this research is to build a foundation of knowledge that can lead both to the construction of usefull legged ~~systems~~ vehicles and to a better understanding of animal locomotion. The report focuses on control of biped locomotion, the use of terrain footholds, running at high speed, biped gymnastics, symmetry in running, and the mechanical design of articulated legs.

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Abstract

This report documents our progress over the past three years in exploring active balance for dynamic legged systems. The purpose of this research is to build a foundation of knowledge that can lead both to the construction of useful legged vehicles and to a better understanding of animal locomotion. In this report we focus on the control of biped locomotion, the use of terrain footholds, running at high speed, biped gymnastics, symmetry in running, and the mechanical design of articulated legs:

- Planar Biped—Control principles originally developed for one-legged hopping were extended to control biped running. A planar biped machine uses this approach to run with an alternating gait, to hop on one leg, and to switch between gaits.
- Rough Terrain—The ability to place the feet on specific footholds is essential to locomotion on rough terrain. We have explored three methods for controlling the length of the stride in order to control foot placement. These techniques allow the planar biped to negotiate obstacles and climb stairs.
- Top Running Speed—Several parameters can limit the running speed of a legged system. Among them are the strength, length, and stiffness of the legs, the range of joint motion, and the actuator force-velocity characteristics. In the course of experimenting with these parameters, the planar two-legged robot has reached a top speed of 5.9 m/s (13.1 mph).
- Biped Gymnastics—The planar biped has done front flips and aerials. The control program that produces flips uses open-loop control patterns in conjunction with the algorithms for normal running.
- Trot, Pace, Bound—We have used a generalization of one leg algorithms to control trotting, pacing, and bounding in a four-legged running machine. One set of control programs generates the three gaits. The machine can also perform transitions between some of the gaits.
- Articulated Legs—We expect folding legs, those that use rotary joints, to be better than telescoping legs. They will have lower moment of inertia, less unsprung mass, a larger range of motion, better ruggedness, and will be easier to build. Tests on a simple folding leg indicate that it has superior mechanical characteristics, but it is more difficult to control.
- Passive Dynamic Running—We are interested in the possibility of designing legged systems whose intrinsic mechanical behavior is very close to the desired behavior. We

have simulated a passive legged system composed of springs, masses, and linkages that runs passively when supplied with suitable initial conditions.

- Internal Combustion Actuators—A typical mobile hydraulic power supply consists of an engine, pump, drivetrain, and actuators. Can the combustions that occur in the engine be moved to the actuator, thereby eliminating the engine, pump, and drivetrain?
- Zero Gravity Running—It is possible to run in the absence of gravity if the legged system travels between two rebound surfaces. We have explored zero-gravity running for the case of a planar one-legged, two-footed machine. The one-g balance algorithms are effective in zero-g without modification.

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Chapter 1

Introduction and Summary

This is the sixth in a series of progress reports describing research on the dynamics of legged locomotion. The work has been done in the Leg Laboratory, located at Carnegie-Mellon University from 1981–1986, and at the Massachusetts Institute of Technology from 1987–present. Our premise is that active balance and dynamics are important for the control of legged systems, robots and animals alike. Systems that balance actively can use footholds that are widely separated or erratically placed, and they can locomote when support is discontinuous in time. They can move along narrow paths where a broad base of support is not available. Dynamics is already a key ingredient in the behavior of animals and it will weigh heavily in the development of useful legged vehicles.

A dynamic treatment need not be an intractable treatment. We have found that simple algorithms can provide balance and control for a variety of dynamic legged systems. The machines we have studied so far include a planar one-legged hopping machine, a three-dimensional one-legged hopping machine, a planar biped running machine, a quadruped, a monopod with a rotary leg joint, and a zero-gravity one-legged two-footed running machine. The techniques used to control each of these machines derive from a single set of control algorithms, modified in various ways. Techniques based on symmetry and the three-part decomposition of control have been used to control a series of laboratory robots that span a diverse set of configurations—one leg, two legs, four legs, telescoping leg, articulated leg—and a range of locomotion behaviors—hopping, pronking, biped running, trotting, forward flip. The ability of simple algorithms to operate under diverse circumstances suggests their fundamental nature.

1.1 Recent Progress

The remainder of this report is a collection of papers that describes these projects. They are summarized in the following paragraphs.

Table 1–1: Summary of Research at the Leg Laboratory

1982	Planar one-legged machine hops in place, travels at a specified rate, keeps its balance when disturbed, and jumps over small obstacles.
1983	Three-dimensional one-legged machine runs and balances on an open floor.
1983	Simulations reveal passively stabilized bounding gait for quadruped-like model.
1984	Data from cat and human runners exhibit symmetries like those used to control running machines.
1984	Quadruped running machine runs with trotting gait. The one-leg algorithms are extended to control this machine.
1985	Planar biped runs with one- and two-legged gaits and changes between gaits.
1986	Planar biped does flips and aerials.
1986	Planar biped sets new speed record of 11.5 mph.
1987	Quadruped runs with trotting, pacing, and bounding gaits.
1988	Planar biped adjusts stride to place feet on footholds. This allows jumping over obstacles and climbing stairs.
1988	Planar biped runs with longer legs, increasing top speed to 13.1 mph.
1988	Quadruped demonstrates rudimentary ability to change between running gaits.
1988	Simulations show energy conservative running motion for simple one- and two-legged systems.
1988	Computer simulation shows running in zero-g by bouncing between two floors.

1.1.1 Planar Biped Running Machine

The algorithms used to control machines that hop on one leg were extended to control a planar biped, which runs on two legs. The basic approach is for the control system to designate an active leg and an idle leg. Because there is just one active leg at a time, the one-leg algorithms can be used to control the biped's behavior. These algorithms focus on controlling hopping height, forward running speed, and body posture. The idle leg is kept short while it is made ready for the next step. Using this approach, the planar biped runs with an alternating gait or a hopping gait, and can change gaits while running.

In addition to studying locomotion on rough terrain and running at high speed, we used the planar biped to perform gymnastic maneuvers; forward flips and aerials. See figure 1–2. The control program that produces a flip uses open-loop control patterns in conjunction with the algorithms for normal running.

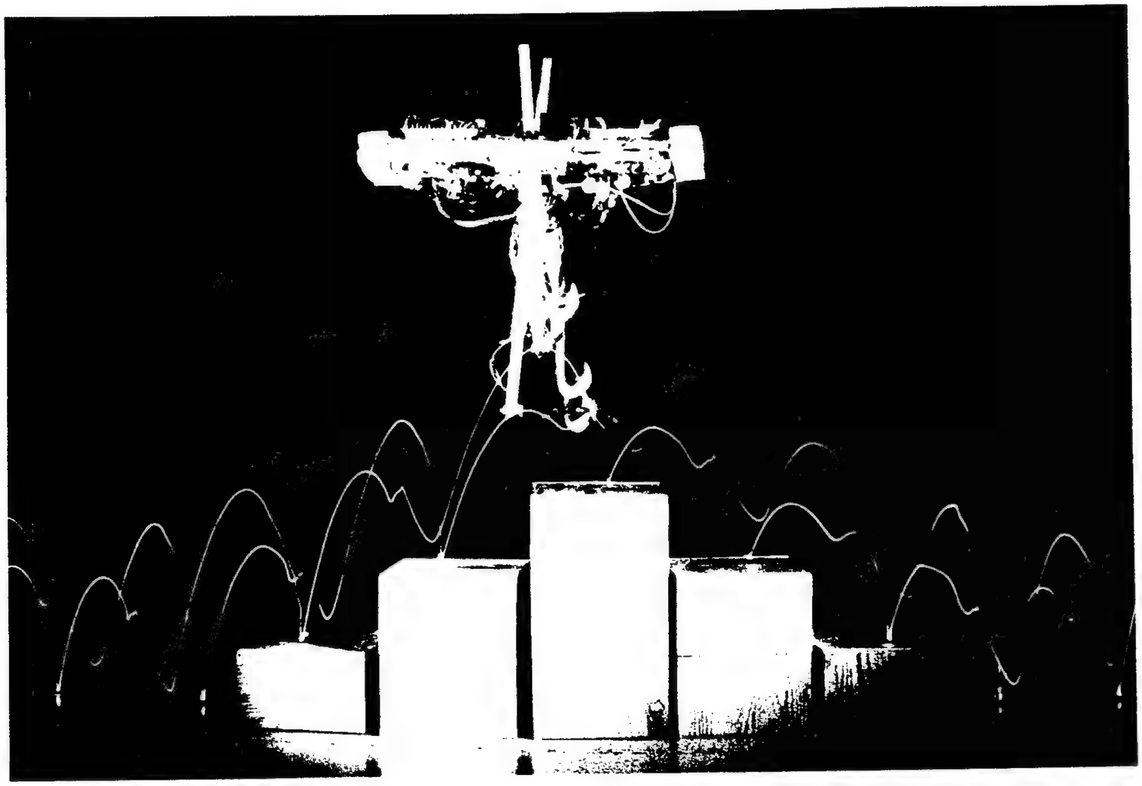


Figure 1–1: Photograph of the planar biped running up and down a short flight of stairs. The machine adjusts the length of each stride as it approaches the stairs, in order to place the feet properly with respect to the steps and during the climb. The machine is shown running from left to right, with the lines indicating the paths of the feet.

1.1.2 Stride Control for Rough Terrain

The promise of travel on rough terrain is the key to the expectation that legged vehicles may someday be useful. Jessica Hodgins is studying locomotion on rough terrain. She addresses the problem of controlling the length of each stride to position the feet on particular footholds on the ground. She has explored three approaches to manipulating stride. One approach adjusts the duration of the flight phase, holding the duration of stance and the forward speed constant. A second approach adjusts the stiffness of the leg to change the duration of the stance phase, holding the duration of flight and forward speed constant. The third approach adjusts forward running speed, holding cadence constant. A practical system running on rough terrain may eventually combine these and other techniques for manipulating stride. For now we are studying them in isolation to understand them better. Using these methods to control stride, the planar biped has climbed stairs, jumped over obstacles, and jumped through a hoop.

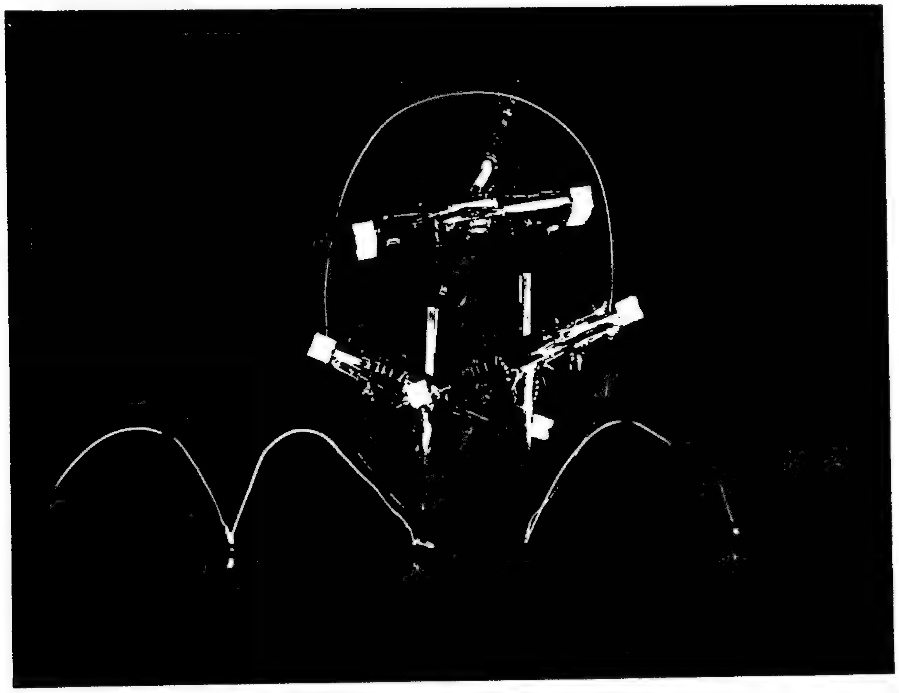


Figure 1–2: Three images of the planar biped as it does a flip. The flash was synchronized with liftoff, peak altitude during flip, and touchdown. The machine ran from right to left.

1.1.3 Fast Running

What are the basic factors that limit speed in legged locomotion? What limits top speed for the planar biped? Energy, control, mechanical design, and computing are broad candidates. We are trying to achieve greater running speed not only because the quest for speed is exciting, but because learning how to make machines run fast motivates the improvement of our understanding of running machines and of running in general. Jeff Koechling has identified physical and control constraints that can limit the running speed of the biped. These efforts have resulted in a 25% increase in top running speed for the planar biped, which now stands at 5.9 m/s (13.1 mph).

1.1.4 Articulated Legs

How can we build legs that are stronger, lighter, faster, and more reliable? One idea is to use rotary joints rather than linear telescoping joints. In previous work we built four machines that ran successfully on telescoping legs. However the mass and moment of inertia of these legs is high, their reliability is low, and they are difficult to build. We believe that articulated legs, those that use rotary joints, can be designed to solve some of these problems. One

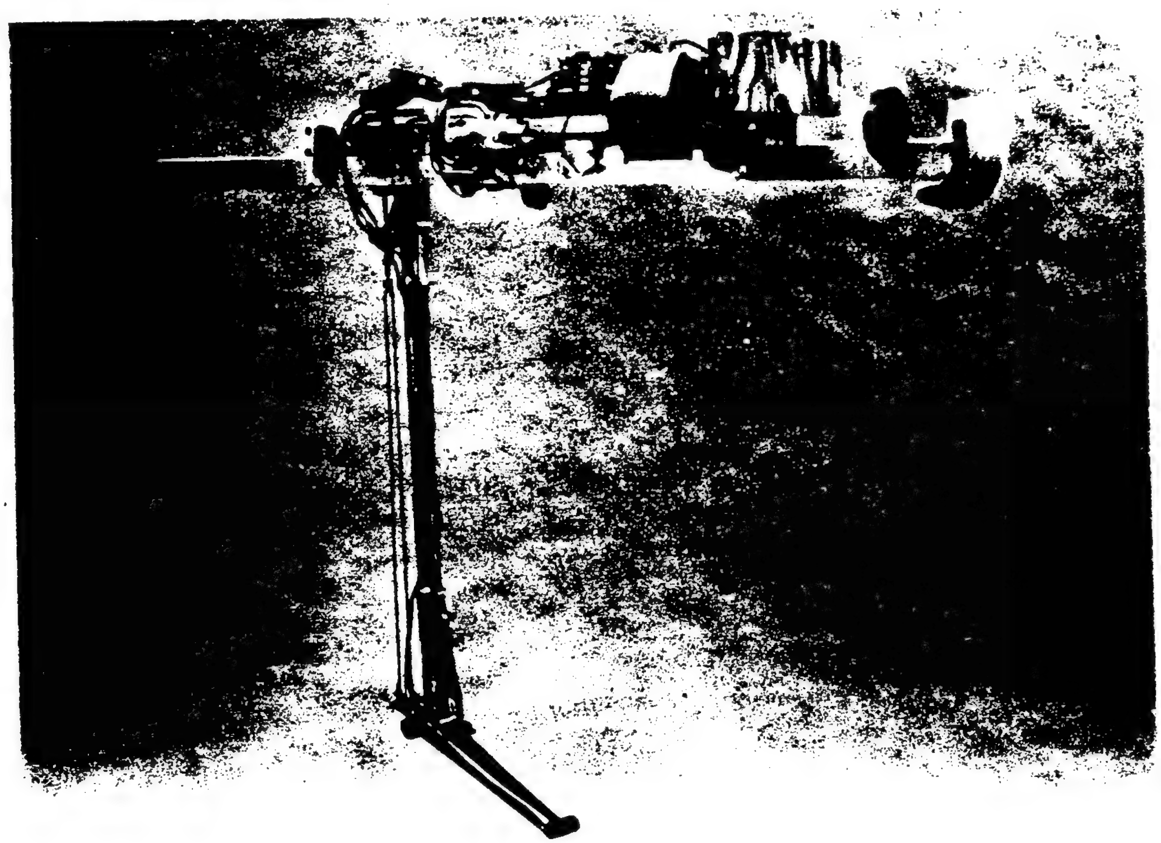


Figure 1–3: Photograph of monopod with articulated leg. The foot is a fiberglass leaf-spring. Hydraulic actuators drive the hip joint and pulls on a tendon attached to the foot.

hurdle will be to incorporate the elastic storage elements vital to good dynamic behavior, without overcomplicating the control. A first design is shown in figure 1–3. The tests we have done show that it performs quite well as part of a planar hopping machine, though its asymmetry and high natural frequency pose new locomotion control problems. This work is aimed at designing an articulated leg for a quadruped.

1.1.5 Passive Dynamic Running

The legged robots we studied previously use springs in their legs to drive the vertical motion of the body. A springy leg allows the vertical motion to be based on a spring-mass oscillation that recycles energy from one step to the next. Can a springy hip be used to improve the efficiency of the legs' fore and aft motions too? To explore this question we have done computer simulations of a simple hopping machine that has no actuators: one spring in the leg allows vertical oscillations while a second spring in the hip allows the leg to swing fore-aft in a passive oscillation. By tuning the mechanical parameters of the system, we have found

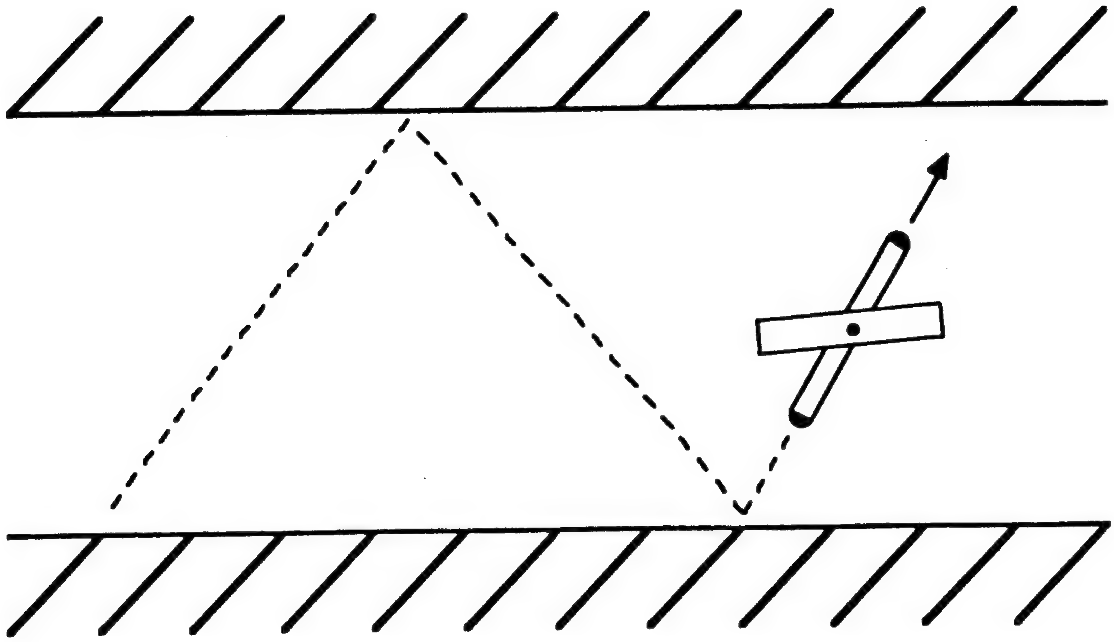


Figure 1–4: Zero gravity running. The pattern of motion exhibited by a system running between two parallel walls.

reentrant trajectories for the system that coordinate the vertical body motions with the leg sweeping motions, and that accommodate the ground interaction constraints. Whereas physical implementations of passive dynamic running will require a source of control for stability and a source of energy to make up for losses, good passive behavior will result in substantial reductions in the energetic cost of locomotion, especially at high speeds.

1.1.6 Internal Combustion Actuator

A typical self-contained power supply for a hydraulic robot includes a gas tank, an internal combustion engine, a linkage, a hydraulic pump, plumbing, servovalve, and a piston driven actuator. Can the internal combustions that occur in the engine be moved to the actuator, so we can eliminate the tank, engine, linkage, and pump? To do so would require a degree of control over the combustions that is not found in any existing engine. Furthermore, the output impedance of the combustion actuator would have to be matched to the input impedance of the robot linkage. If these problems can be solved, the result would be a self-contained system with high power to weight ratio. So far we have built a hydrogen powered internal combustion actuator that operates on the workbench. Running machines that use internal combustion actuation are in the early planning stage.

1.1.7 Zero Gravity Running

Normally it is not possible to run without gravity, because the upward motion that initi-

ates the flight phase can not be reversed once contact with the ground is lost. One way to overcome this problem is to run between two floors. In this case, the supporting forces generated in the collision with one floor reverse the vertical velocity generated during collision with the other floor. Such a configuration for running might exist in a space station, where the walls could act as a pair of rebound surfaces. To test running between two walls, we simulated a planar machine with one leg and two feet. See figure 1–4. We found that one-leg algorithms function well in the simulated zero-g environment.

The work reported in this document focuses on principles of legged systems, including their mechanical design, approaches to sensing and control, and the computations needed for effective locomotion. Progress in developing these principles can lead to better understanding of animal locomotion and to the construction of useful legged vehicles. We also believe that progress in legged locomotion can contribute more generally to the development of dynamic robots, and to other forms of physical systems.

This is the final report for the three-year DARPA contract MDA903-85-K-0179, which ended August 1988. A complete bibliography of work done in the Leg Laboratory can be found at the end of this document.

Chapter 2

Planar Biped Locomotion

2.1 Introduction

Bipedal locomotion is a behavior that humans and animals perform with agility, grace, and speed, but robots have not yet mastered. In this paper we examine bipedal locomotion by exploring the control and coordination of a two-legged laboratory robot. Our approach takes advantage of the fact that a biped frequently run with a gait that uses one leg for support at a time. For such a gait, the behavior of the support leg can be controlled as though it were the only leg in the system, while the other leg is considered idle and held immobile with respect to the body. This approach allows algorithms used previously to control one-legged hopping to be extended to two-legged running.

The algorithms for hopping on one leg consisted of separate control laws used to regulate hopping height, body attitude, and running speed. These algorithms were adequate to provide dynamic balance for planar and three-dimensional one-legged hopping machines (Raibert and Brown 1984, Raibert, Brown and Chepponis 1984). To extend the algorithms for bipedal running, we added a bookkeeping mechanism so that the legs would take turns providing support, and a controller for the idle leg to ensure that it would not collide with the ground.

We built a planar bipedal robot to test the approach to bipedal running. We simplified the problem by constraining the biped to move in a plane, and by using an off-board computer and power supply. Once operational, the machine ran in place, traveled at specified speeds, maintained balance when disturbed, and changed gaits between running and one-legged hopping. The one-leg hopping algorithms were adequate to control biped running, but we found that performance could be improved by causing the idle to mirror the motions of the active leg, 180 degrees out of phase. This tail-like use of the idle leg reduced the pitching motions of the body.

After a brief review of previous work on biped locomotion, we review the algorithms used

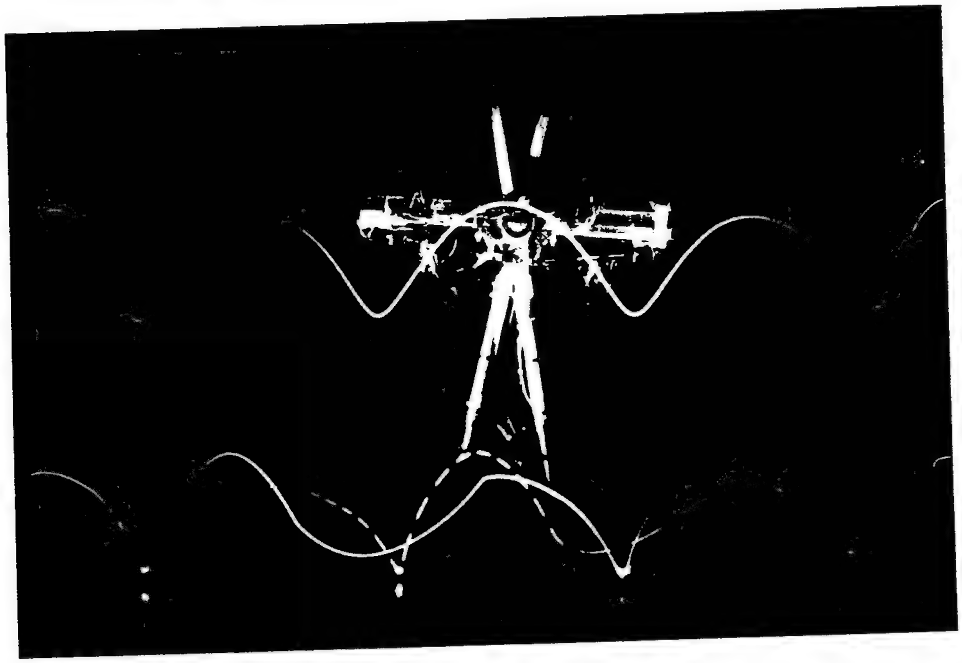


Figure 2–1: Photograph of the planar biped. Light sources were attached to the feet and the center of the body. The dashed line is a flashing light source on leg 1, which is the leg farther from the camera. The biped ran from left to right in the darkened laboratory. The control computer triggered the flash when the machine was directly in front of the camera. The trajectory of the light sources illustrates the bouncing motion of the machine.

to control one-legged hopping machines and describe the modifications needed for bipedal running. Then we describe the biped apparatus used for laboratory experiments and present data that characterize the machine's operation. We close with a discussion of how the biped running algorithms have been extended to allow more advanced bipedal behavior, including fast running, running on rough terrain, and simple gymnastic maneuvers.

2.1.1 Background

Kato and his colleagues built an early computer controlled biped (Ogo et al. 1980). Their biped had ten hydraulically powered degrees of freedom that moved two large feet. The first version of their machine was statically stable, moving along a preplanned trajectory that kept the center of mass of the body located over the base of support provided by the grounded foot. Each step took several seconds.

A later version of Kato's machine used a dynamic tipping phase to transfer support from one foot to the other (Kato et al. 1983). This machine was statically stable most of the time. Once during each step, however, the machine slowly leaned forward until the center of mass moved forward of the front edge of the supporting foot. It then tipped forward

onto the other foot, which was positioned so that it would catch the machine and passively return the system to static equilibrium. An inverted pendulum model of the system was used to determine where to place the catching foot.

This approach was an interesting way to achieve dynamic behavior. The system was not dynamic in the sense of reacting at run-time to the progress of the motion. Instead, an off-line analysis of the dynamics of the system specified how to position the catching foot statically to get run-time dynamic behavior. Knowledge of dynamics of the system were *compiled*, if you will, into a simple run-time strategy.

Miura and Shimoyama (1984) built the first walking machine that balanced actively. It adjusted its motions in response to changes in the dynamic state of the system. Their stilt biped, Biper 3, was patterned after a human walking on stilts, with each foot providing only a point of support. The machine had three actuators. One actuator moved one of the legs sideways, one actuator moved the other leg sideways, and a third actuator separated the legs fore and aft. Sensors in each foot detected ground contact and measured the angle of the leg with respect to the ground.

The control for Biper 3 was also derived from the inverted pendulum model of tipping during single support. Unlike the Kato machine, this biped adjusted the placement of each foot in response to the ongoing behavior of the system as measured by the sensors. Separate mechanisms controlled tipping in the forward and sideways directions. In each direction, placement of the foot was adjusted in response to the actual and desired tipping motion. Since the legs could not shorten, the machine lifted each leg by rocking onto the other leg. The machine rocked from one foot to the other, while the airborne foot was repositioned according to the algorithm. Because of the stiff legs, when the machine walked it looked like Charlie Chaplin.

There are many theoretical studies of biped locomotion. These studies can be characterized by the technique that is used to reduce the order of the model and to make the problem tractable. For example, Gubina, Hemami, and McGhee (1974) assumed a massless leg. Vukobratovic and Stepanenko (1973) added extra constraints to the motion of their biped model to resolve the indeterminacies that occur when both feet are on the ground. Furusho and Masubuchi (1987) assumed that the ankle joint of the support leg was passive.

Matsuoka (1979; 1980) was the first to build a machine that ran with periods of ballistic flight. During each flight phase the machine completely lost contact with the ground. He formulated a model consisting of a body and one massless leg and derived a time-optimal state feedback controller that provided stability for hopping in place and for hopping with translation. To test this controller, Matsuoka built a planar one-legged hopping machine that operated in low gravity by lying on a table inclined 10° from the horizontal. The machine hopped about once per second and balanced as it traveled back and forth on the table.

2.2 Review of Hopping on One Leg

Raibert and his colleagues built planar and three-dimensional one-legged hopping machines that hopped in place, traveled along simple paths, jumped over obstacles, and maintained balance when disturbed mechanically (Raibert and Brown 1984; Raibert, Brown, and Chepponis 1984; Raibert 1986). These machines used a simple control system that had independent controllers for hopping height, forward speed, and body attitude. The purpose of their experiments was to focus on the role of balance in legged locomotion, while avoiding the issues of gait and inter-leg coordination. In this section we give a brief description of the one-legged hopping algorithms because they provide the basis for our approach to biped locomotion.

Each one-legged hopping machine had a rigid body and a springy telescoping leg that pivoted with respect to the body at a hinge-type hip joint. One actuator exerted torque between the leg and the body and a second actuator acted along the axis of the leg, in series with a spring in the leg.

The task of controlling the hopping machines was decomposed into three parts. One part sustained the machine's bouncing motion, the second part regulated the angle of the body in space, and the third part stabilized the forward running speed. A summary of the control is as follows:

- *Hopping Height*—Hopping was a resonant bouncing motion of the spring-mass system formed by the springy leg and the mass of the body. A leg actuator excited the motion by thrusting during stance. The hopping converged to a height for which the mechanical losses occurring throughout the hopping cycle were just balanced by the energy added during thrust.
- *Body Attitude*—The control system regulated the angle of the body by applying torques to the body during stance. Vertical loading on the foot kept it from slipping when hip torques were applied. A linear servo moved the body toward its nominal angle whenever the foot was on the ground:

$$\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}) \tag{2.1}$$

  where τ is the hip torque, ϕ is the angle of the body, ϕ_d is the desired angle of the body, $\dot{\phi}$ is the angular rate of the body, and k_p, k_v are gains.
- *Forward Running Speed*—During each flight phase, the control system positioned the foot to control the acceleration of the body during the next stance phase. See figure 2–2. When the control system placed the foot in the center of the distance the body traveled during stance, the forward speed at liftoff was the same as it had been at touchdown. We call this position of the foot the *neutral point*. When the control system displaced the foot from the neutral point, the body accelerated, with magnitude and direction of acceleration proportional to the magnitude and direction of the displacement. The control system displaced the foot from the neutral point by a distance proportional to the difference between the actual speed and the desired speed. The algorithm the

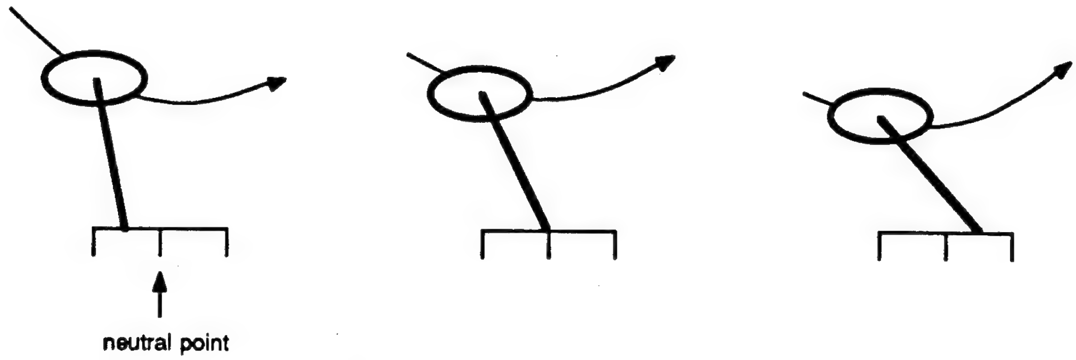


Figure 2–2: Displacement of the foot from the neutral point accelerates the body by skewing the symmetry of the body's trajectory. When the foot is placed closer to the hip than the neutral point, the body accelerates forward during stance and the forward speed at liftoff is higher than the forward speed at touchdown (left). When the foot is placed further from the hip than the neutral point, the body accelerates backward during stance and the forward speed at liftoff is slower than the forward speed at touchdown (right). Horizontal lines under each figure indicate the distance the body travels during stance, and the curved lines indicate the path of the body.

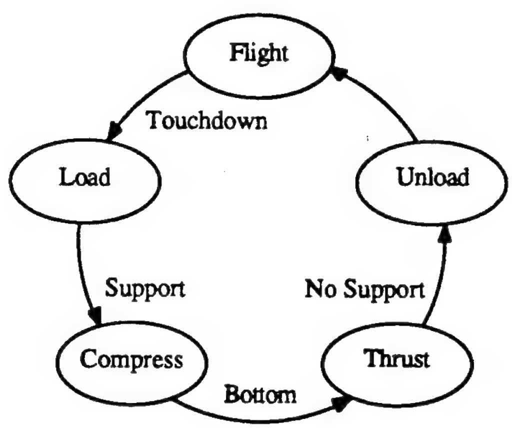


Figure 2–3: State machine for planar one-legged hopping machine. The five states are shown along with the events that trigger the state transitions.

control system used to place the foot was:

$$x_f = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{2.2}$$

wherex_f is the forward displacement of the foot from the projection of the center of gravity, $\dot{x}$ is the forward speed, $\dot{x}_d$ is the desired forward speed, T_s is the predicted duration of the next support period, and $k_{\dot{x}}$ is a gain. The first term of equation 2.2 is an estimate of the neutral point and the second term is the correction for speed errors. Once the control system found x_f, a kinematic transformation determined the hip angle that would position the foot as specified, and a linear servo drove the hip actuator.

The control program used a cyclic state machine to keep track of the behavior of system as it hopped. The state machine specified whether or not each of the three controllers

should operate during each phase of the hopping cycle. These algorithms stabilized the hopping of both a two-dimensional one-legged machine that was mechanically constrained to operate in a plane, and a three-dimensional one-legged machine that traveled freely about the laboratory. More detailed accounts of the control algorithms used for the one-legged machines and of the experimental results can be found in the references.

2.3 Bipedal Running Is Like Hopping

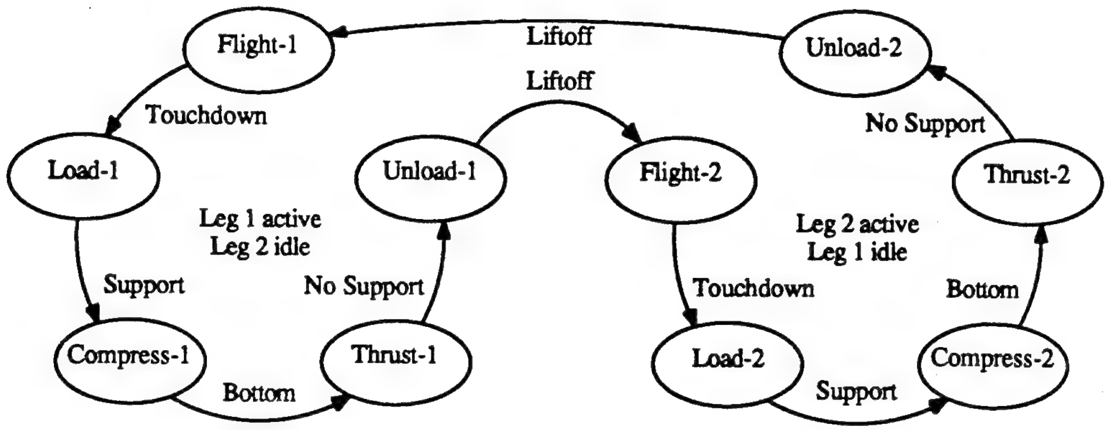
Running bipeds typically use just one leg for support at a time. During running, there is a strict alternation between support phases, during which one or the other leg supports the body by pushing downward on the ground, and flight phases, during which no legs touch the ground. We call the leg providing support or just about to provide support, the *active leg*, and the other leg the *idle leg*.

We argue that the active leg can be controlled using the algorithms developed for the one-legged machines. If the idle leg is kept immobile with respect to the body, then the dynamics of bipedal running are the same as the dynamics of one-legged hopping. Because the biped uses just one leg for support at a time, the vertical thrust delivered by the active leg can be calculated correctly using the algorithm that the one-legged system used. The torque exerted between the active leg and the body to keep the body level can also be calculated using the one-leg algorithm. Finally, because just one of the biped's legs is placed on the ground at a time, the algorithm for calculating the placement of the active leg can be the same algorithm as used by one-legged systems. Thus the control system for a biped can use the same three algorithms for controlling hopping height, body attitude, and forward running speed as were used previously by the control system for a one-legged hopping machine.

Despite these similarities, there are several differences in the control of one- and two-legged running. One difference is that a two-legged system needs the ability to shorten and lengthen its legs substantially. The idle leg must shorten so that it does not strike the ground while the active leg is compressed during stance, and it must lengthen again in preparation for its turn providing support, soon after it becomes the active leg. The mechanical design of the leg must enable these motions.

Another difference is in the sequence of states that occur during running. The state machine for a biped must keep track of which leg is active and which is idle, and switch between them at appropriate times. The composite state machine shown in figure 2–4 performs this function. It is essentially two copies of the state machine used for the one-legged systems, joined together so that the legs take turns providing support. At liftoff, the legs switch roles with the idle leg becoming active and the active leg becoming idle.

These two changes extend the one-legged control algorithms for bipedal running. A third modification of the algorithms, called mirroring, improves stability and reduces pitching of the body. Mirroring causes the hip motion of the idle leg to follow the hip motion of the active leg, but with opposite sign. This allows the net angular momentum of the system



State	Trigger Event	Actions
FLIGHT	Active leg leaves ground (liftoff)	Interchange active and idle legs Lengthen active leg for landing Position active leg for landing Shorten idle leg Mirror angle of active hip with idle hip
LOADING	Active leg touches ground (touchdown)	Zero active hip torque Keep idle leg short Mirror angle of active hip with idle hip
COMPRESSION	Active leg air spring shortens (support)	Servo pitch with active hip Keep idle leg short Mirror angle of active hip with idle hip
THRUST	Active leg air spring lengthens (bottom)	Extend active leg Servo pitch with active hip Keep idle leg short Mirror angle of active hip with idle hip
UNLOADING	Active leg air spring approaches full length (no support)	Shorten active leg Zero hip torques active leg Keep idle leg short Mirror angle of active hip with idle hip

Figure 2–4: Finite state machine that coordinates two-legged running. The state shown in the left column is entered when the event listed in the center column occurs. The controller advances through the states in the sequence indicated by the arrows. The LOADING and UNLOADING states occur when the foot is on the ground, but the leg spring is not compressed. To avoid skidding the foot along the ground, no hip torque is applied in these states.

to remain small when the legs sweep back and forth, without large body pitching motions.

2.4 Planar Biped Experiments

We built a planar two-legged running machine to test the control of bipedal running using the one-leg algorithms. The machine, shown in figure 2–1, has two telescoping legs connected to the body by pivot joints at the hips. A hydraulic actuator exerts a torque about each hip, between the leg and the body. A hydraulic actuator within each leg works in series with a pneumatic spring. Together, they change the length of the leg and make the leg compliant along its long axis.

A tether boom mechanically constrains the machine to move on the surface of a large sphere. Locally, the machine can move fore and aft, up and down, and pitch nose up or nose down. Figures 2–5 and 2–6 show the details of the planar biped machine. The appendix gives the physical parameters and the machine's kinematics. A computer program records data from the machine's sensors, executes the control algorithms, and servos the actuators.

Constrained by its tether boom, the biped runs in a circle in the laboratory. It steps alternately on each foot as it moves up and down with a rhythmic bouncing motion. The paths of the feet and the vertical bouncing motion of the body are illustrated by figure 2–1. The top graph of figure 2–7 shows the vertical motion of the biped as it ran with an alternating gait. The two legs operated 180° out of phase, as shown by the middle graph. During each support phase the leg spring first compressed and then extended, as shown in the final graph.

A bipedal running machine has more opportunities to regulate the angular momentum of the body than does a machine with only one leg. During flight, a machine hopping on one leg must swing its leg forward to position it for the next support phase. The hip torques that swing the leg forward also pitch the body forward, so the body accumulates a pitch error that must be corrected during the stance phase.

A machine with two legs can swing the legs, keep the body level, and conserve angular momentum, all at the same time. This can be done by moving the two legs with equal and opposite motions, so that the hip torques exerted on the body to move one leg just cancel the hip torques exerted on the body to move the other leg. We call this approach *mirroring*, because the motion of the idle leg is made to mirror the motion of the active leg. Mirroring is not perfect, because the two legs do not have the same length all the time, so their moments of inertia are not always equal. To reduce body rotations during hopping, a kangaroo's tail mirrors the motion of its legs.

The effects of mirroring the legs is illustrated in figure 2–8. For this experiment, the planar biped hopped on one leg, with the idle leg servoed to a fixed angle with respect to the body. At the time indicated by the vertical dashed line, the hip angle of the idle leg was servoed to the negative of the hip angle of the active leg. This mirroring of the legs reduced the maximum pitch oscillations of the body by about a factor of two.

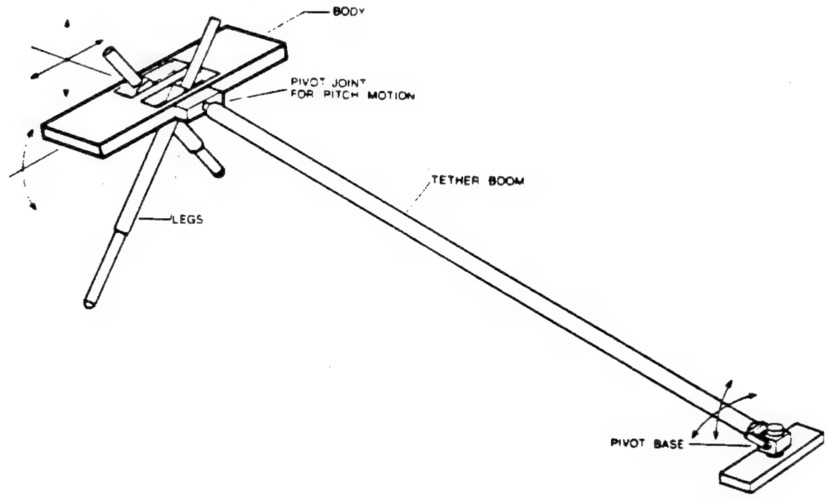


Figure 2–5: Planar biped running machine used for experiments. The body is an aluminum frame, on which are mounted hip actuators and computer interface electronics. Each hip has a low friction hydraulic actuator that positions the leg fore and aft. An actuator within each leg changes its length, while an air spring makes the leg springy in the axial direction. Figure 2–6 shows the leg in greater detail. Sensors measure the lengths of the legs, the positions and velocities of the hip actuators, pressures in the leg air springs, contact between the feet and the floor, and the pitch angle of the body. An umbilical cable connects the machine to hydraulic, pneumatic, and electrical power supplies, and to the control computer, all of which are located nearby in the laboratory. The arrangement of body, legs, hips, and actuators provides a means to control the position of the feet with respect to the body, to generate an axial thrust with each leg, and to provide hip torques during running. The tether boom constrains the machine to motion in two dimensions, fore and aft, up and down and rotation in the plane, and it provides a means of sensing body pitch angle and vertical and horizontal position in the room.

The planar biped can run with the alternating gait described above, it can run by hopping on one leg, and it can switch between the two gaits. It runs with a hopping gait by making one leg active all the time, and the other idle. The bouncing motion of the machine is unchanged when running on one leg and, if mirroring is turned on, the idle leg acts like a tail. It is easy to do a transition from one gait to the other by switching between state machines at the beginning of the flight phase. Figure 2–9 shows data recorded as the biped switched from two-legged running to one-legged hopping and back again, as it traveled forward at 2.8 m/s.

2.5 Fast Running, Rough Terrain, and Gymnastics

We have used the bipedal locomotion algorithms just described as the basis for other legged behaviors. We have done experiments in which the planar biped ran at high speed, traveled

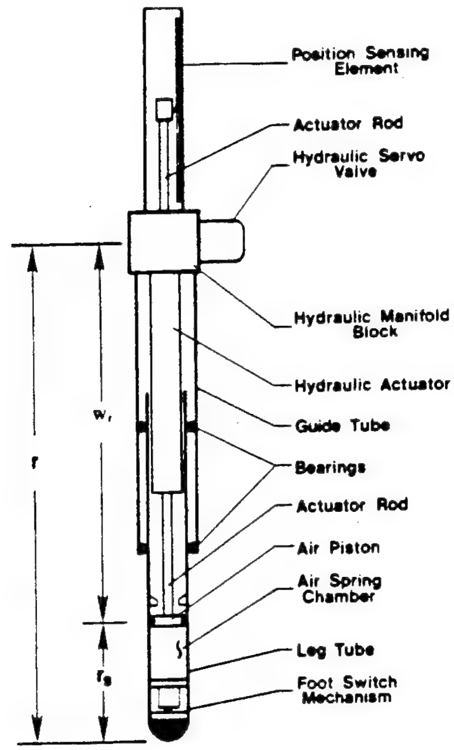


Figure 2–6: Diagram of leg used in running machine. A hydraulic actuator acts in series with an air spring. The hydraulic actuator is used to drive resonant bouncing motion of the machine and to retract the leg during flight. It also acts in conjunction with the air spring to determine the axial force the leg exerts on the ground. Sensors measure hydraulic actuator length, overall leg length, air pressure in the spring, and loading on the foot.

over simple forms of rough terrain, and performed gymnastic maneuvers. In each of these cases we used the algorithms described earlier, either as a substrate on which to build, or as a starting point from which to modify.

In the course of studying the limits of running speed in legged locomotion, Koechling made the planar biped run fast (Koechling and Raibert, 1988; Koechling 1989). In the fastest experiment, the machine traveled at 5.9 m/s (13.1 mph) for over 40 m. To achieve top speed, the legs of the planar biped were lengthened by 0.18 m, the control algorithms for body attitude were modified compensate for the pressure/flow characteristics of the hip actuators, and the leg springs were made about twice their normal stiffness. Aside from these changes, the control algorithms for high-speed running were same as the algorithms described earlier.

Hodgins (1988, 1989) studied how an actively balanced legged system could adjust the length of each of its steps without losing balance. The ability to adjust step length is an important component of traveling on rough terrain. She found that the control system could adjust the length of each step by manipulating the duration of the stance phase, the duration of the flight phase, or the forward running speed. In the course of her experiments, Hodgins programmed the planar biped to step on a desired target, jump over obstacles, and

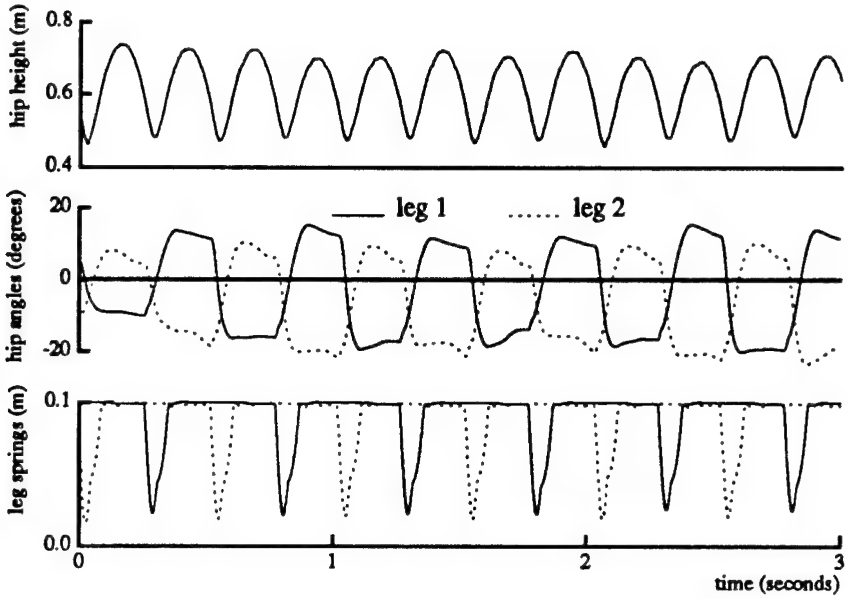


Figure 2–7: Running data for the planar biped. The top curve shows the height of the hip above the floor. The middle curve shows the angles of the two legs with respect to the body in the fore-aft plane. The legs oscillate 180° out of phase and at half the frequency of the bouncing motion. The bottom curve shows compression of the air springs as the legs are used for support in alternation. (TL.226.14)

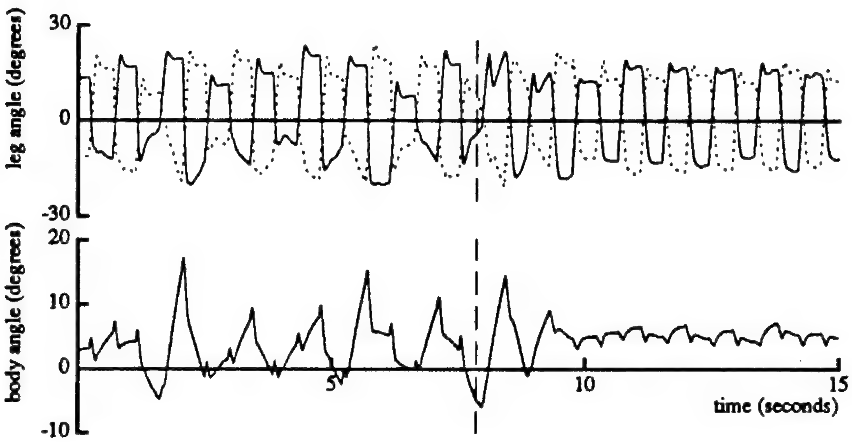


Figure 2–8: A control algorithm that kept the leg angles equal and opposite reduced the the amplitude of the oscillations in body attitude. The top graph shows the angle of each leg with respect to the axis of symmetry of the body. The vertical line marks a switch from an algorithm that moved the legs independently to one that ensured that the leg angles were mirror images. The axis of symmetry was a line passing through the hip joint perpendicular to the body. The bottom graph shows that mirroring reduced the oscillations in body angle from about 20° peak-to-peak to about 6° peak-to-peak. In this experiment the planar biped was running about 2.5 m/s (5.6 mph). (Data file B87.325.3)

to run up and down a short flight of stairs. Figure 1–1 shows the climb and descent of the

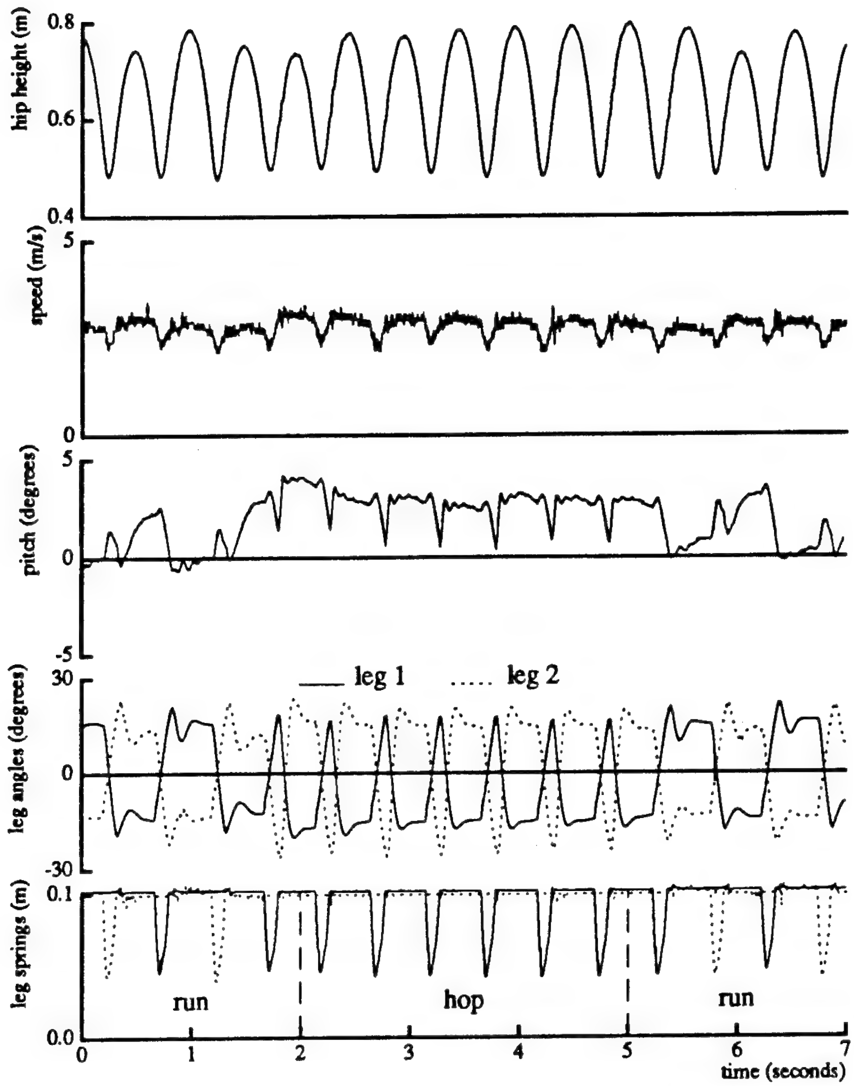


Figure 2–9: Gait transitions. The planar biped switched from two-legged running to one-legged hopping at 2 seconds. It switched back at 5 seconds. The transition did not disturb the forward speed, body pitch, or vertical bouncing. (B89.123.2)

stairway.

To study the production of discrete maneuvers and to have some extra fun, we made the planar biped do a forward flip (Hodgins and Raibert, 1987) See figure 2–10. To perform a flip the biped machine runs forward, brings the legs together and thrusts with both legs to jump high, exerts a large hip torque to pitch the body forward, shortens the legs to tuck once airborne, untucks in time to land on the feet, and then continues running. To develop this behavior, we modified the algorithms that operate during three steps of otherwise normal running.

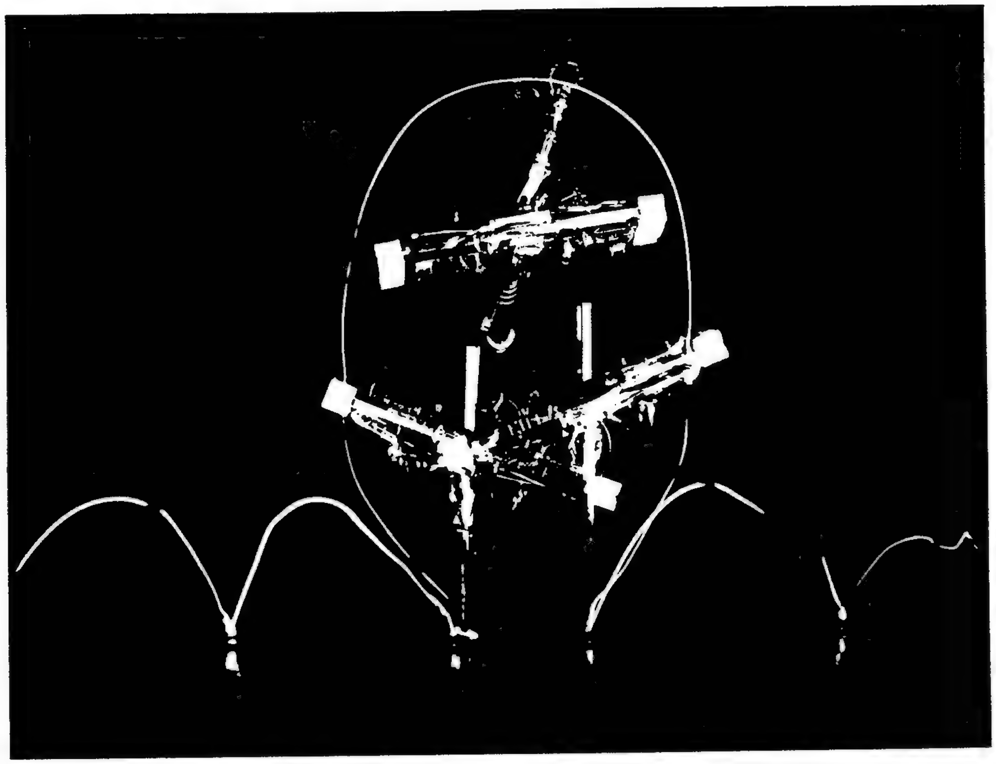


Figure 2–10: Photograph of planar biped doing a flip. Three flashes were synchronized with liftoff, the top of flight, and touchdown. The machine was running from right to left, with a light indicating the path of the near foot. The tether boom is hidden behind the body.

2.6 Summary

There is a class of gaits called the one-foot, for which only one foot touches the ground at a time and each stance phase is separated by a flight phase. A biped executing a one-foot gait can be controlled with algorithms developed for one-legged hopping machines. One of the two legs is designated the active leg, which is used to adjust hopping height, body attitude, and forward speed. The other leg is designated the idle leg, which is kept short to clear the ground and servoed to mirror the sweeping motions of the active leg. A state machine synchronizes the actions of the control computer to the behavior of the running machine, and selects which leg will be active on each step.

We have demonstrated the feasibility of this approach with a planar two-legged running machine that operates in the laboratory. It runs with an alternating gait, a one-legged hopping gait, and it can switch between gaits. The planar biped and the basic control algorithms have also been used to investigate the limitations of fast running, the control of step length for rough terrain, and simple robot gymnastics.

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2.8 Appendix A: Physical Parameters of Planar Biped Running Machine

Body:		
length	0.75 m	(30 in)
width	0.23 m	(9 in)
mass	11.5 kg	(25 lb_m)
moment of inertia	0.4 kg-m^2	(1370 lb_m-in^2)
Leg:		
total mass	1.66 kg	(3.66 lb_m)
unsprung mass	0.29 kg	(0.64 lb_m)
moment of inertia	0.13 kg-m^2	(444 lb_m-in^2)
Hip actuator:		
Bore	0.01613 m	(0.625 in)
Rod diameter	0.00953 m	(0.375 in)
Area	1.27×10^{-4} m^2	(0.197 in^2)
Stroke	0.051 m	(2.0 in)
Hip sweep	± 0.52 radian	($\pm 30°$)
Maximum velocity	0.500 m/s	(19.7 in/s)
Maximum force	2630 N	(591 lbf)
Moment arm	0.0444 m	(1.75 in)
Maximum Torque	117 N-m	(1030 in-lb_f)
Leg actuator:		
Bore	0.0127 m	(0.500 in)
Rod diameter	0.00953 m	(0.375 in)
Area	5.54×10^{-5} m^2	(0.0859 in^2)
Stroke	0.23 m	(9.0 in)
Maximum velocity	3.42 m/s	(11.21 ft/s)
Maximum force	1146 N	(258 lb)
Maximum leg length	0.67 m	(26.4 in)
Minimum leg length	0.44 m	(17.3 in)
Air spring:		
Bore	0.0286 cm	(1.125 in)
Area	6.42×10^{-4} m^2	(0.994 in^2)
Length	0.10 m	(4.0 in)
Stiffness (50 psi, full length)	7500 N/m	(43 lbf/in)
Hip spacing	0.090 m	(3.54 in)
Boom radius	2.54 m	(100 in)
Circle circumference	15.96 m	(628 in)
Computer	DEC VAX 11-785	

2.9 Appendix B: Kinematics of Planar Biped Running Machine

The planar biped has nine degrees of freedom, listed in table 2–1. We refer to the biped as a planar machine, because it has three external degrees of freedom. The tether boom actually constrains the hip to move on the surface of a sphere. The boom pivots at the base, allowing the machine to run around the circle and to jump up and down. A swivel joint at the end of the boom closest to the machine allows the machine to rotate about the in the plane of the running motion. Figure 2–11 shows the external degrees of freedom.

Planar Biped Degrees of Freedom

degree of freedom	range of motion
horizontal position, x	0–16 m
vertical position, z	0.4–1.4 m
pitch, ϕ	±180°
leg lengths, ℓ_1, ℓ_2	0.44–0.67 m
leg actuator positions, h_1, h_2	0.00–0.23 m
hip actuator positions, w_1, w_2	±0.025 m

Table 2–1. The planar biped has nine degrees of freedom. A potentiometer senses the position along each degree of freedom. The length of each leg spring is determined by the difference between the total leg length and the position of the leg actuator: $s = \ell - h - 0.338\,\mathrm{m}$.

The position of each foot with respect to the hip is determined by the length ℓ and angle θ of the leg:

$$x_{fh} = \ell \sin(\theta)$$
$$z_{fh} = -\ell \cos(\theta)$$

as shown in figure 2–12. The leg angle in turn depends on the pitch angle ϕ of the body, and the angle γ between the leg and a line perpendicular to the body:

$$\theta = \gamma - \phi.$$

Figure 2–13 illustrates the kinematics of the hip. Lengthening or shortening the hydraulic actuator changes the angle of the leg with respect to the body, moving the foot fore and aft. Each millimeter of hip actuator motion moves the foot between ten and fifteen millimeters, depending on the leg length.

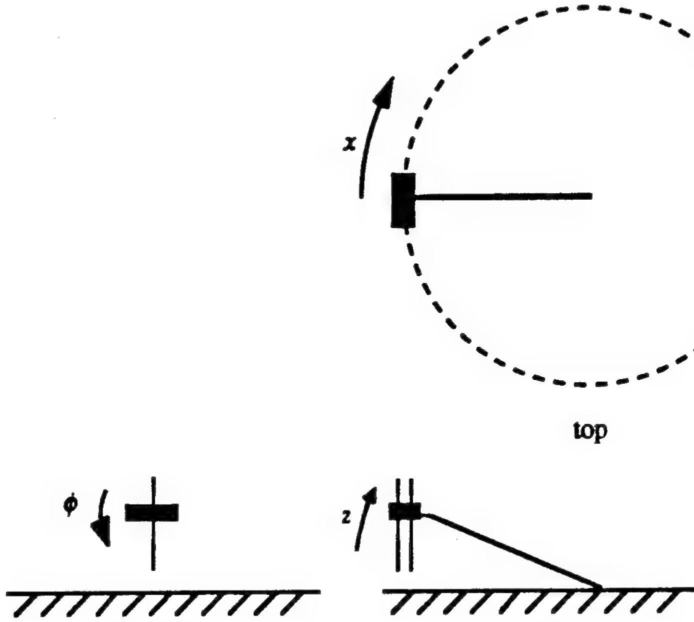

Figure 2–11: The boom constrains the biped's motion to the surface of a sphere. The boom pivots at the base, allowing the machine to run around the circle and to jump up and down. There is also a pivot joint at the machine end of the boom, which allows the machine to rotate about its pitch axis. These three degrees of freedom allow the machine to move fore and aft, up and down, and to rotate in the plane.

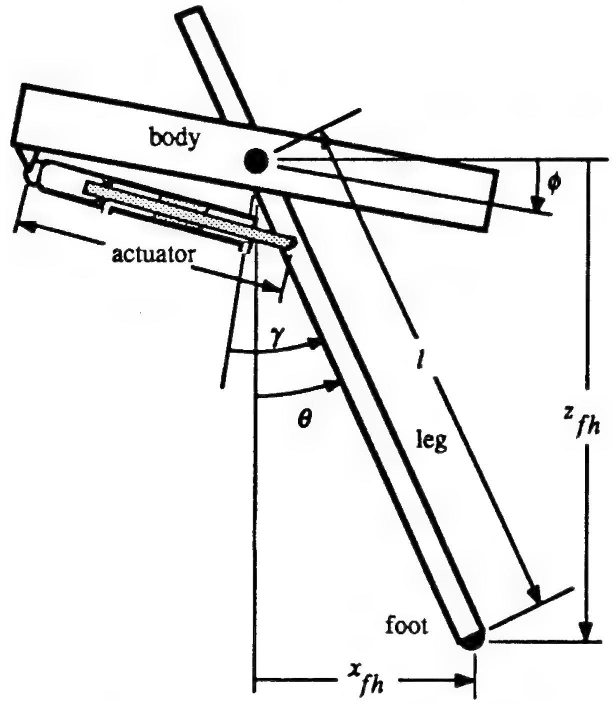

Figure 2–12: Kinematics of planar two-legged running machine. The length of the leg is ℓ, the angle between the leg and vertical is θ and the pitch angle of the body is ϕ. $\theta = \gamma - \phi$. The horizontal foot position relative to the hip, $x_{fh} = \ell \sin(\theta)$.

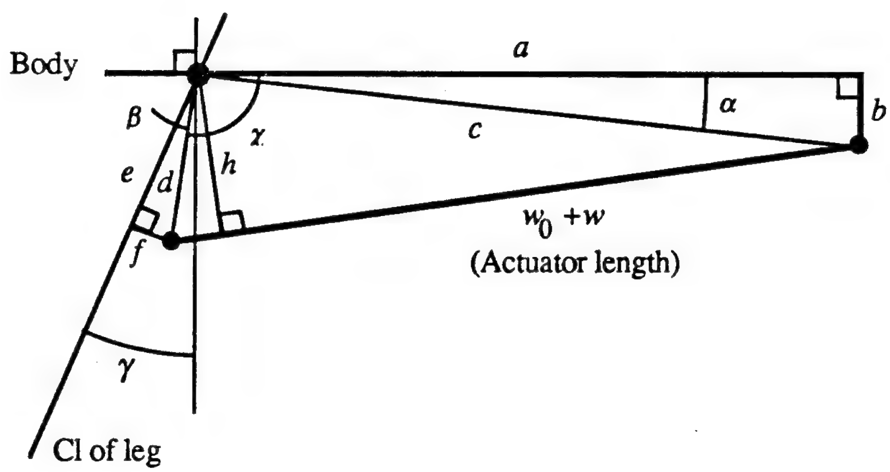

Figure 2–13: This diagram shows the attachment points of the hip actuator to the leg and to the body. The parameters are: $a = 0.3194\,\text{m}$, $b = 0.0032\,\text{m}$, $e = 0.0439\,\text{m}$, $f = 0.0062\,\text{m}$, and $w_0 = 0.316\,\text{m}$. Solving triangles abc and def gives: $c = 0.3194\,\text{m}$, $\alpha = 0.0994$ radian, $d = 0.0444\,\text{m}$, and $\beta = 0.140$ radian. Solving triangle cdg, where $g = w_0 + w$, gives: $\chi = \arccos\left\{\left[c^2 + d^2 - (w_0 + w)^2\right]/2cd\right\}$. The hip angle is $\gamma = \alpha + \beta + \chi - \pi/2$, and the moment arm is $h = cd\sin(\chi)/(w_0 + w)$. The the hip actuator range of motion is $0.025\,\text{m} < w < 0.025\,\text{m}$, giving hip angles of -0.58 radian $< \gamma < 0.59$ radian.

Chapter 3

Adjusting Step Length for Rough Terrain Locomotion

3.1 Abstract

Animals use their legs to take them to every corner of the Earth. They travel on rough terrain by using isolated footholds and by stepping onto or over obstacles. In order for legged robots to travel on rough terrain, they too must be able to use available footholds while keeping their balance and avoiding unsuitable terrain. Rough terrain requires legged systems to sense the terrain, plan paths, choose sequences of footholds, and adjust the length of each step to place the feet on the chosen footholds. As a first step towards understanding these abilities in legged robots, we have studied the control of step length. We have explored three methods for controlling step length in the context of an actively balanced planar biped running machine. The planar biped has used these methods to place its feet on specified targets, to leap over obstacles, and to run up and down stairs.

3.2 Introduction

Legged vehicles may someday travel on terrain that is too rough for wheeled and tracked vehicles of comparable size. To do so they will have to use whatever footholds are available, even those which are isolated or occluded by obstacles. The task of traveling over rough terrain includes many sub-problems, including terrain sensing, path planning, selections of foothold, and adjustment of step length. This paper concentrates on the last of these problems, the ability to adjust the length of each step so as to place the feet on chosen footholds.

To study this problem we have considered a special case of rough terrain locomotion, in which the footholds are unevenly spaced on a straight line in the horizontal plane. Motion

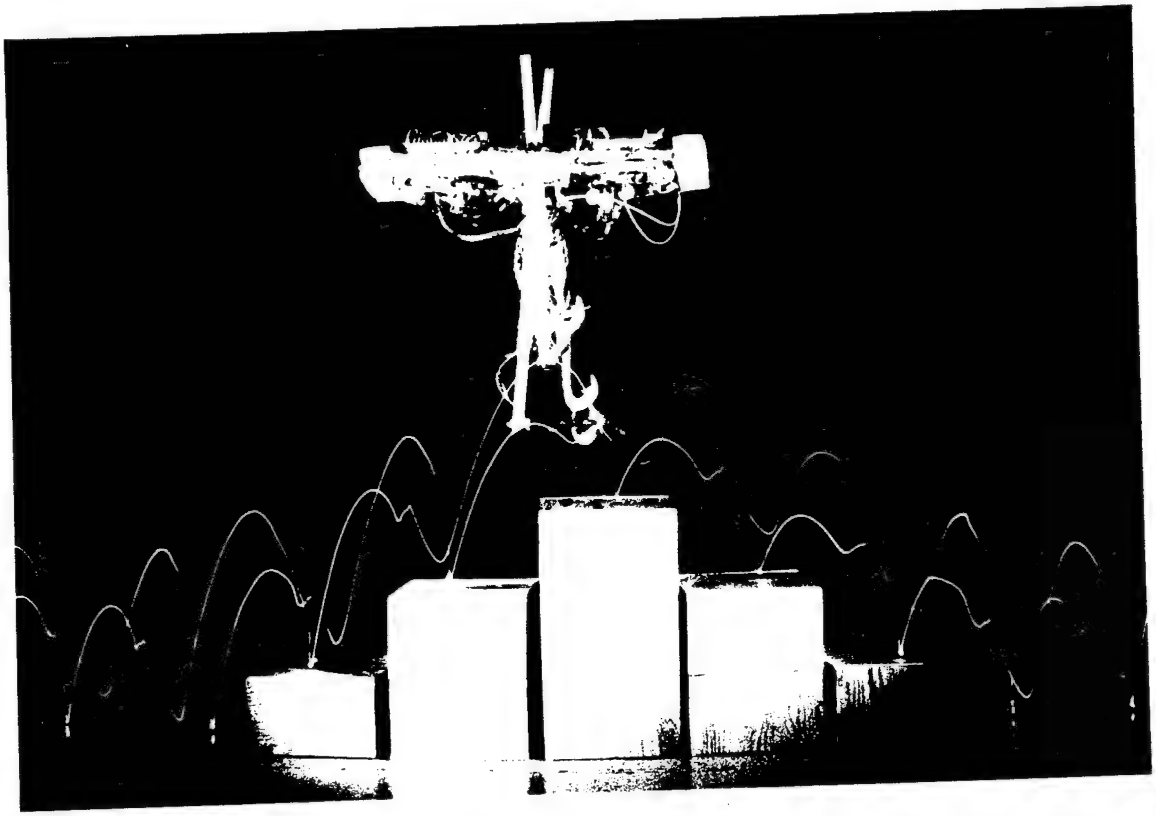

Figure 3–1: Photograph of the planar biped running up and down a flight of three stairs. The control system adjusts the length of the machine's steps so that the feet land approximately in the center of each stair. The machine is shown running from left to right. Light sources indicate the paths of the feet.

of the legged system is confined to the vertical plane that contains the footholds. We have considered systems that run rather than walk, that balance themselves actively, and that use one foot for support at a time. This paper discusses three methods for achieving control of step length under these circumstances, and describes laboratory evaluation of the methods using a planar biped running machine.

In this paper we consider three methods for controlling the step length used by a legged system as it runs and balances. One method adjusts the forward running speed, keeping flight and stance duration constant. The second method adjusts the duration of the flight phase by manipulating the vertical thrust delivered to the body during the stance phase. The third method adjusts the duration of the stance phase by manipulating the stiffness of the leg spring on which the body rebounds. We also consider placing the foot directly on the desired target, without taking balance into account. Through experiments on a planar biped running machine we find that all three methods are able to provide changes in the step length, but the forward running speed method gives the widest range of adjustments and good accuracy.

After a brief review of previous work on rough terrain locomotion, we describe the ma-

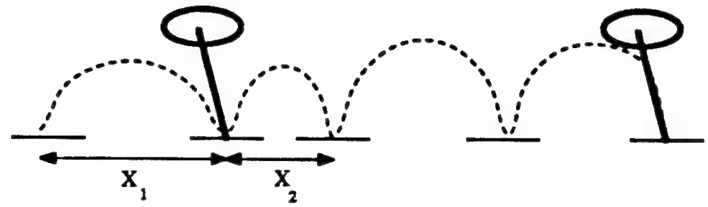

Figure 3-2: One-dimensional rough terrain consists of a series of footholds with uneven spacing in the direction of running. The footholds lie on a straight line at the intersection of the horizontal and sagittal planes. The legged system is constrained to move in the sagittal plane. Three-dimensional rough terrain would include vertical and lateral variations in the spacing of footholds.

chine we used for our experiments and the details of the methods studied for adjusting step length. Data from the experiments are presented, along with a comparison of their precision and relative strengths and weaknesses. We close with a discussion of demonstrations that require adjustments in step lengths, placing a foot on a target, jumping over obstacles, and climbing stairs.

3.3 Background

Lee, Lishman, and Thomson (1982) studied skilled long jumpers, who must place their feet near the front edge of the takeoff board if they are to maximize their jump. They found that subjects manipulated the vertical impulse delivered to the ground by the legs. Vertical impulse, the integral of vertical force exerted on the ground during the stance phase, determines the duration of the flight phase, and, assuming constant speed forward travel, it determines the length of each step. A series of adjustments in step length permitted the jumpers to arrive at the takeoff board with the toe very near the leading edge. The use of vertical impulse to control step length is quite similar to the flight duration method for controlling step length that we report below.

Warren, Lee, and Young (1985) studied how runners adjust step length when required to place their feet on randomly positioned footholds on a treadmill. They were primarily interested in the use of vision for placing the feet on visible targets. They obtained results that confirm the long jump results, in that the subjects used vertical impulse to control step length, with nearly constant forward speed.

The first walking machines to walk on rough terrain used fixed patterns of gait generated by kinematic linkages (Morrison 1968; McGhee 1976). These legged vehicles had no feedback and could not adapt to variations in the terrain, but they sometimes could successfully walk over obstacles.

The quadruped transporter, built by Ralph Mosher and his colleagues at General Electric, walked on rough terrain with a human providing control and sensing (Mosher 1968; Liston 1964; Liston 1970). The human drove the machine by making crawling motions

with his arms and legs. A hydraulic force-reflecting master-slave servo that caused the four legs of the vehicle to follow the motions of the operator, and transmitted force information from the legs of the vehicle back to the arms and legs of the operator. Despite the intense concentration required to drive the machine, Mosher was able to make it amble along at about 5 mph, climb a stack of railroad ties, and walk through an orange grove. These experiments showed that a human made legged machine could move effectively on rough terrain, provided it has an excellent control system.

Hirose (1984) built a quadruped which used a set of reflexes to walk on rough terrain. One reflex pulled the foot back and lifted it, if a sensor on the foot indicated that it had bumped into an obstacle. Another reflex caused the support legs to push downward if a load cell in the foot indicated that it was not bearing an adequate load. A third reflex caused the vertical lengths of the legs to be adjusted so the body remained level, as indicated by a pendulum. Hirose's quadruped used these reflexes to climb over objects and up and down steps without a model of the terrain.

Other work on legged robots moving on rough terrain has primarily addressed the task of choosing footholds (Okhotsimski and Platonov 1973; Okhotsimski and Platonov 1975; McGhee and Iswandhi 1979; Gurfinkel et al. 1981; Devjanin et al. 1983; Ozguner, Tsai, and McGhee 1984; Hirose 1984). These studies assume slow-moving statically stable systems, for which the task of placing the feet on the chosen footholds is accepted as simple.

The control of step length is not simple for dynamically stable systems, or for those that move with moderate speed. The length of each step in a dynamic system is determined by forward speed, duration of the flight phase, and duration of the stance phase, none of which can be adjusted instantaneously. The task requires the control system to coordinate the forward and vertical velocities of the system with the motions of the legs. The actions taken to produce a desired step length must be accomplished while maintaining stability. For legged systems that balance actively, adjustment of step length is a more highly constrained task than the control of normal running.

3.4 Control of Step Length

The length of a step is the distance between two successive footholds, as illustrated in figure 3–3. This distance consists of three parts: the distance traveled during the second half of one stance phase ($1/2\dot{x}_sT_s$), the distance traveled during the flight phase ($\dot{x}_fT_f$), and the distance traveled during the first half of the second stance phase ($1/2\dot{x}_sT_s$), where $\dot{x}_s$ and $\dot{x}_f$ are the forward speed during the stance and flight phases, and T_s and T_f are the duration of the stance and flight phases. For steady-state running, $\dot{x}_s$ and T_s are constant and $L_{step} = \dot{x}_sT_s + \dot{x}_fT_f$. The length of a stride is the distance between two successive touchdowns of the same foot. For a one-legged machine, the length of the step is equal to the length of the stride. For a biped running with an alternating gait, the length of the stride is twice that of the step. In the biomechanics literature, "step length" refers to the distance traveled by the body while the foot is on the ground, but that is not the definition

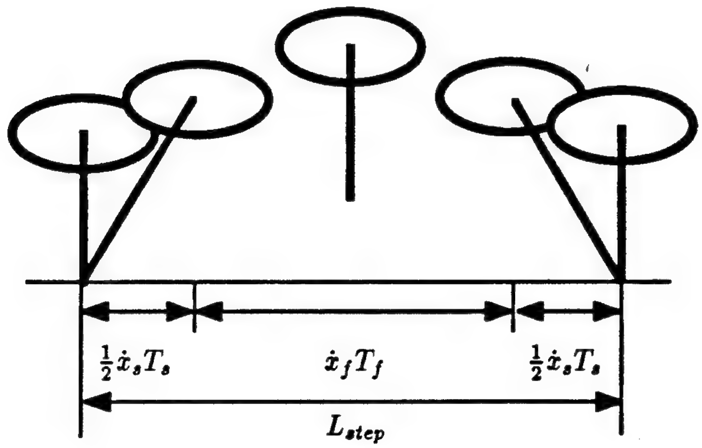

Figure 3–3: The step length is broken into three parts: the distance traveled during the second half of one stance phase, the distance traveled during flight, and the distance traveled during the first half of a second stance phase.

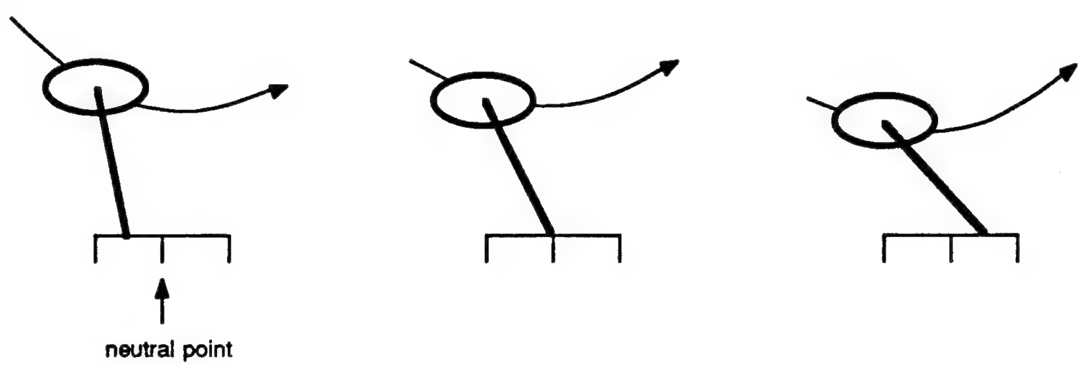

Figure 3–4: Controlling forward speed. Center) When the foot is placed in front of the hip one-half the distance the body will travel while the foot is on the ground, the forward speed will remain unchanged. We call that location the ***neutral point***. Left) When the foot is displaced backward from the neutral point, the system accelerates forward. Right) Displacing the foot forward from the neutral point causes the system to decelerates. See (Raibert 1986) for details.

we use in this paper.

The distance traveled by the body during a given period is the product of the duration of the period and the forward speed. Therefore, variations in the duration of the flight phase, duration of the stance phase, or the forward speed will each influence the step length. These observations suggest three methods for controlling step length while maintaining balanced running:

- *Forward Speed*—For given durations of the stance and flight phases, forward speed determines step length. The control system manipulates the forward speed of the

system by positioning the foot to accelerate or decelerate the system on each step, as described in Raibert and Brown (1984). The control system can position the foot to cause zero, positive, or negative net acceleration for the stance phase, as shown in figure 3–4. The forward position of the foot at touchdown (x_{fh}) is determined by

$$x_{fh} = \frac{T_s \dot{x}}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{3.1}$$

where T_s is a prediction of the duration of the next stance phase, $\dot{x}$ is the forward speed, $\dot{x}_d$ is the desired forward speed, and $k_{\dot{x}}$ is an empirically determined gain. The placement of the foot at touchdown is determined by an estimate of the center of the neutral point $(1/2T_s\dot{x})$ and a correction term for errors in the forward speed $(k_{\dot{x}}(\dot{x}-\dot{x}_d))$.

The task of controlling step length by manipulating forward speed requires that the forward speed both be controlled accurately and that the system respond quickly to changes in desired forward speed. To achieve this level of performance we improved the model of the behavior during stance and reduced pitch disturbances. These improvements lead to a second equation for the position of the foot at touchdown:

$$x_{fh} = k_{np} T_s \frac{k_s(\dot{x} + \dot{x}_d)}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{3.2}$$

where k_{np}, k_s, and $k_{\dot{x}}$ are empirically determined gains.

- *Flight Duration*—With constant forward speed, the duration of the flight phase determines the distance traveled during flight. The duration of the flight phase is determined by the vertical velocity of the body when the foot leaves the ground and the difference between the altitude of the body at liftoff and touchdown. If the altitude of the body is the same at touchdown and liftoff, then the length of time the system will spend in flight is

$$T_f = \frac{2\dot{z}_{lo}}{g} \tag{3.3}$$

where $\dot{z}_{lo}$ is the vertical velocity at liftoff, and g is the acceleration of gravity. The control system manipulates the amount of thrust so that the energy in the system at liftoff will produce the correct flight duration. The desired energy is a function of the desired flight duration:

$$E = \left(z_{lo} + \frac{gT_f^2}{8}\right)mg \tag{3.4}$$

where z_{lo} is the vertical altitude of the body at liftoff and m is the mass of the system. Because energy is converted between its potential and kinetic forms, the measure of energy at any time during stance predicts the duration of the flight phase. The control system monitors the energy throughout stance and adds energy by extending the actuator in the leg. The resulting thrust makes up for mechanical losses and accommodates changes in the duration of the flight phase from one step to the next.

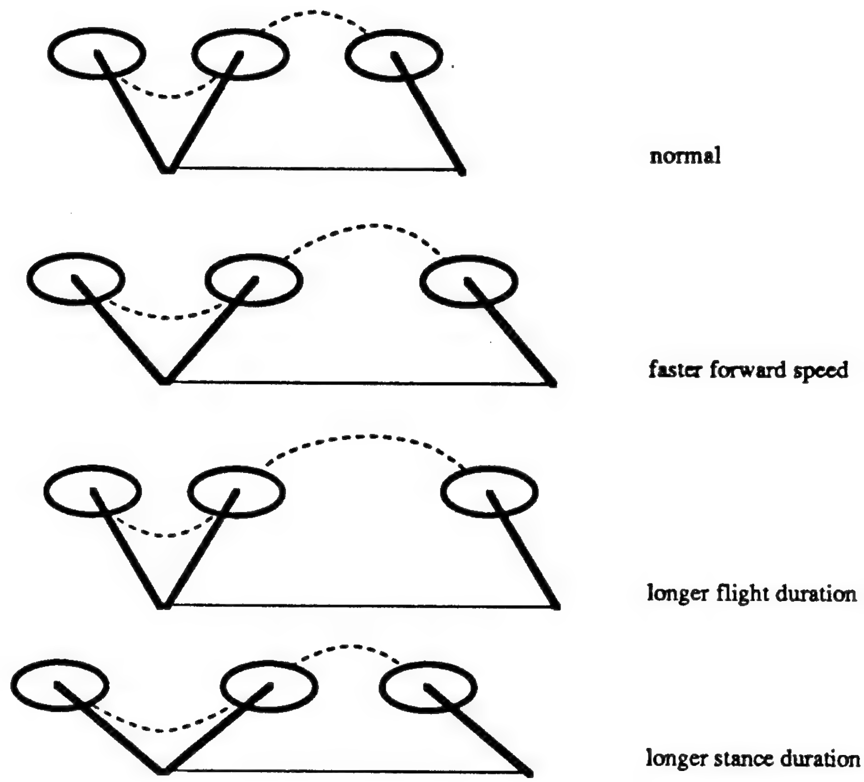

Figure 3–5: Three methods for controlling step length. The top drawing portrays a normal step. The others show longer steps produced by the adjusting one of the parameters of the step. The second drawing has increased forward speed, the third an extended flight phase, and the fourth an extended stance phase. In each case increasing one of the parameters of the step produces a longer step length.

- *Stance Duration*—The distance the body travels during the stance phase is the product of the average forward speed and the duration of the stance phase. The duration of the stance phase is determined, to first order, by the mass of the body and the stiffness of the leg. During stance, the hopping machine is a spring-mass oscillator. An approximate natural frequency, ω_0, is given by $\sqrt{k/m}$, where k is the stiffness of a linear approximation to the leg's air spring and m is the mass of the body supported by the leg. The control system manipulates the stiffness of the leg by controlling the air pressure in the air spring during flight. Higher pressures cause the spring to be stiffer and reduce the stance duration of the system. The control system uses this manipulation of stance duration to control the distance traveled during stance.

The three methods are illustrated in figure 3-5. Each of these methods manipulates one variable to produce a desired step length, leaving the other two unchanged. The best control of step length will probably be achieved through some combination of the three methods, by manipulating several variables at a time. However, in the interests of understanding the

effects of each manipulation, we allowed only one variable to change at a time. Knowledge about the performance of each method in isolation should aid in developing good strategies for combining the methods.

Each of these methods for adjusting step length allows the placement of the foot at touchdown to be calculated using the normal running algorithms. In normal running, placement of the foot at touchdown is used both to maintain balance and to regulate forward speed. We call the foothold chosen by the normal algorithms the *balance foothold.* In the context of a dynamic legged system, the task of adjusting step length involves moving the balance footholds to coincide with the desired terrain footholds. All three methods for control of step length adjust parameters so that the balance footholds move to the desired terrain footholds.

An alternative would be to ignore the balance footholds and place the foot directly on the desired terrain foothold. This approach would provide foot placements of high precision. However, to the extent the balance foothold and the desired terrain foothold are not coincident, placing the foot on the terrain foothold will disturb the system's balance and forward speed on subsequent steps. In some cases, the system will regain its balance and in other cases it may tip over.

If one of the three methods for controlling step length described earlier can be used to move the balance foothold close to the desired terrain foothold, then direct placement might be used to make the final small correction that places the foot on the desired terrain foothold with precision. The separation between the balance foothold and the actual placement would be small enough for the system to continue running. Such a technique might fail when the task involves a long series of precisely specified footholds.

3.5 Experiments

Experimental Apparatus

To study the control of step length, we used a planar, two-legged running machine for experiments. Figures 3–6 and 3–7 illustrate the design of the planar biped machine. It has two telescoping legs connected to the body by pivot joints that form hips. Each hip has a hydraulic actuator that positions the leg fore and aft. An actuator within each leg changes the leg length, while an air spring makes the leg springy in the axial direction. The biped is constrained mechanically to move fore and aft, up and down, and to rotate about the pitch axis of the body.

To develop the control algorithms used for adjusting step length, we modified and extended a set of control programs used previously to make the planar biped run. The general approach in the previous algorithms was to decompose the control into three parts. One part regulated the amplitude of the machine's bouncing motion, the second part maintained

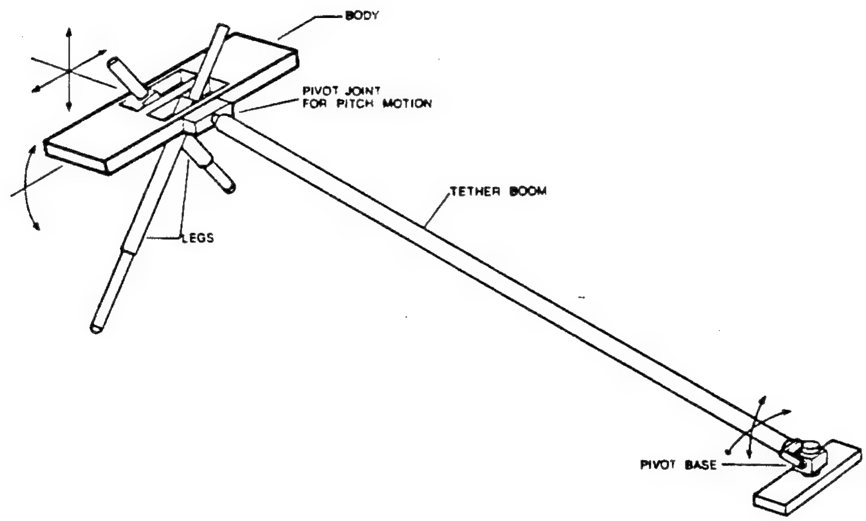

Figure 3–6: Diagram of planar, two-legged running machine used for experiments. The body is an aluminum frame on which are mounted hip actuators and computer interface electronics. Each hip has one low friction hydraulic actuator that positions the leg fore and aft. An actuator within each leg changes its length, and an air spring makes the leg springy in the axial direction. Sensors measure the lengths of the legs, the positions and velocities of the hip actuators, pressures in the air springs, contact between the feet and the floor, and the pitch angle of the body. An umbilical cable connects the machine to hydraulic, pneumatic, and electrical power supplies, and to the control computer, all of which are located nearby in the laboratory. The arrangement of body, legs, hips, and actuators provides a means to control the position of the feet with respect to the body, to generate an axial thrust with each leg, and to provide hip torques during running. A tether boom constrains the machine to move fore and aft, up and down, and to rotate about the pitch axis. The tether boom also provides a means of sensing body pitch angle and vertical and horizontal position in the room.

the body in an upright posture, and the third part controlled the forward running speed. Details of the control system for the planar biped are given in Hodgins, Koechling, and Raibert (1985). These control algorithms adjust speed and hopping height, but they do nothing to specify the length of the step nor the positions on the ground where the feet are to be placed. In this paper we focus on extending the approach to provide control over the length of each step.

Controlling step length using one of the methods described above requires the solution of two problems. First, the controlled parameter must be correctly adjusted. Forward speed, flight duration and stance duration must each be accurately controlled. And second, the adjustment in the controlled parameter must result in the correct step length. The next three sections describe experiments in controlling step length using these three methods.

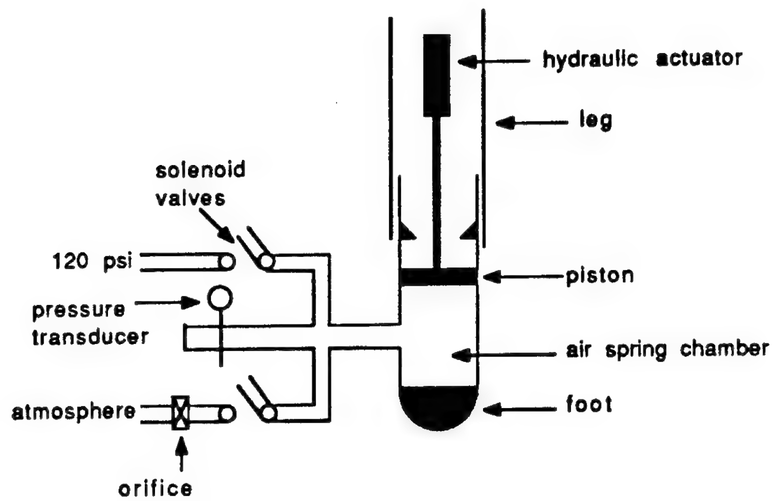

Figure 3–7: Diagram of the leg used in the planar biped machine. A hydraulic actuator acts in series with a pneumatic spring. The hydraulic actuator drives the vertical bouncing motion of the machine, and retracts the leg during flight. Two-way solenoid valves regulate the flow of air to the chambers of the spring. Sensors measure hydraulic actuator length, overall leg length, pressure in the spring, and contact between the foot and the ground.

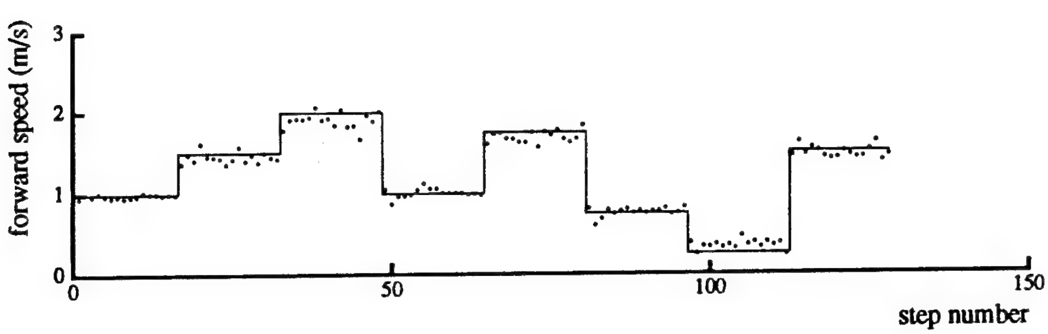

Figure 3–8: Desired and actual running speed. The solid line is the desired forward speed and the dots represent the forward speed actually generated. The biped ran for sixteen steps at each desired forward speed and then the desired speed changed to the next value in the pattern. The average error in forward speed was 0.06 m/s. The stance duration was 0.14 s and the flight duration was 0.47 s. (B87.248.4)

Forward Speed

During each flight phase, the control system manipulates the forward speed by choosing a forward position for the foot that accelerates the body properly during the next support interval. Figure 3–8 shows a pattern of desired forward speeds (solid line) and the forward

Figure 3–9: Desired forward speed is manipulated to adjust step length. The desired step length is changed every eight steps. The solid line is the desired step length and the dots represent the step length that was actually generated. The desired step lengths range between 0.1 m and 1.1 m and the average error in step length was 0.07 m. The flight duration was 0.4 s and the stance duration was 0.15 s for this experiment. (B88.102.0)

speed that was actually achieved (dots). The forward speed ranged between 0.25 m/s and 2.0 m/s with an average error of 0.06 m/s.

By choosing the forward speed correctly the control system can manipulate the step length. Figure 3–9 shows the biped following a pattern of step lengths using adjustments in forward speed to produce the desired step length. The range of step lengths was between 0.1 m and 1.1 m and the average error was 0.07 m.

Duration of Flight

The duration of the flight phase can be adjusted by manipulating the energy in the system at liftoff. Figure 3–10 shows the results of an experiment designed to test the ability of the control system to regulate flight duration. The biped ran forward while the control system attempted to achieve a range of desired flight durations. Flight durations between 0.2 s and 0.5 s were achieved. The average error in flight duration was 0.03 s.

Adjustments in the duration of flight can be used to control step length as shown in figure 3–11. The biped ran forward at 1.1 m/s while the control system varied the flight duration to achieve a pattern of desired step lengths. The average error in step length was 0.07 m when the desired step length was within the range of step lengths possible through adjustments in flight duration.

Duration of Stance

The control system manipulates the stiffness of the leg spring to control the duration of stance. The resting pressure of the spring determines its stiffness. The biped can run with air pressures between 10 psi and 100 psi, and stance times between 0.1 s and 0.2 s. Figure 3–12 shows the results of an experiment designed to test the ability of the control system to

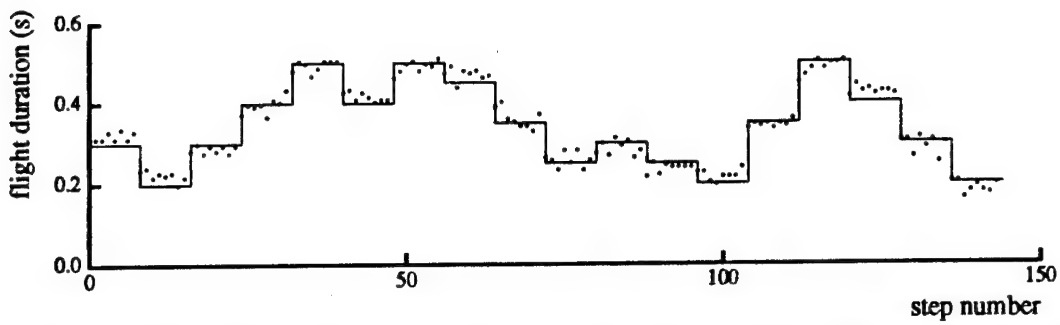

Figure 3–10: Thrust is manipulated to adjust the duration of the flight phase. The solid line is the desired flight duration and the dots are the flight durations that were achieved. The average error in flight duration was 0.03 s. $\dot{x} = 1.2\,\mathrm{m/s}$, $T_s = 0.15\,\mathrm{s}$. (B88.336.12)

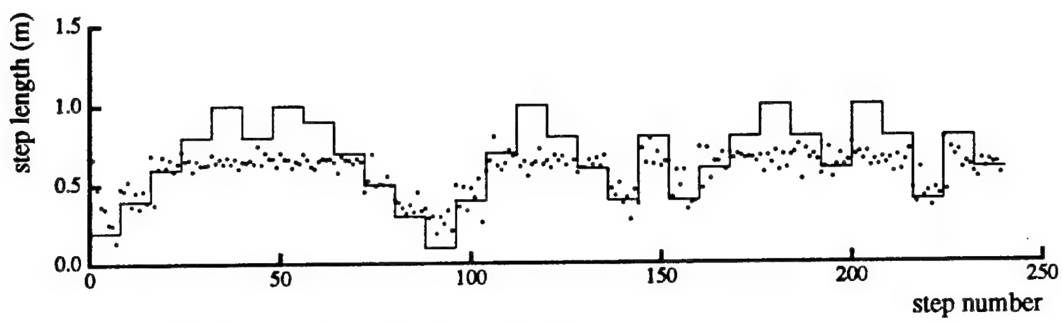

Figure 3–11: Desired flight duration is manipulated to adjust step length. Desired step length, (solid line) is changed every eight steps. The desired step lengths range between 0.1 m and 1.1 m. The average error in step length was 0.07 m when the desired step length was within the range of step lengths possible through adjustments in flight duration. $\dot{x} = 1.1\,\mathrm{m/s}$, $T_s = 0.15\,\mathrm{s}$. (B88.106.10)

regulate stance duration by adjusting the resting pressure of the air spring. The biped ran forward at 1.0 m/s while the control system attempted to achieve a range of desired stance durations. The desired stance durations were between 0.1 s and 0.2 s. The average error in measured stance duration was 0.005 s.

Figure 3–13 shows the results of using adjustments in stance duration to control step length. The biped ran forward at 1.1 m/s while the control system varied the stance duration to produce a pattern of desired step lengths. The average error in step length was 0.03 m when the desired step length was within the range of step lengths possible with adjustments in stance duration. This pattern of step lengths was much narrower than the pattern of step lengths used to test the other two methods because the range of step lengths produced by adjustments in stance duration is small.

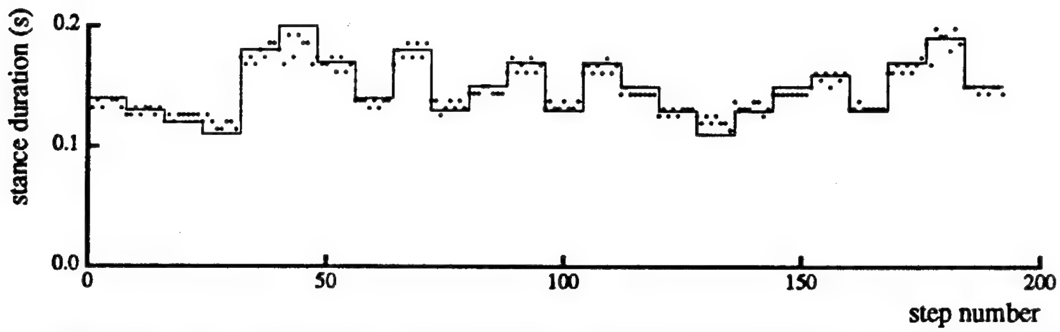

Figure 3–12: Leg stiffness is manipulated to control stance duration. The solid line is the desired stance duration and the dots are the measured stance durations. The average error in stance duration was 0.005 s. $\dot{x} = 1.0\,\mathrm{m/s}$, $T_f = 0.38\,\mathrm{s}$. (B88.243.4)

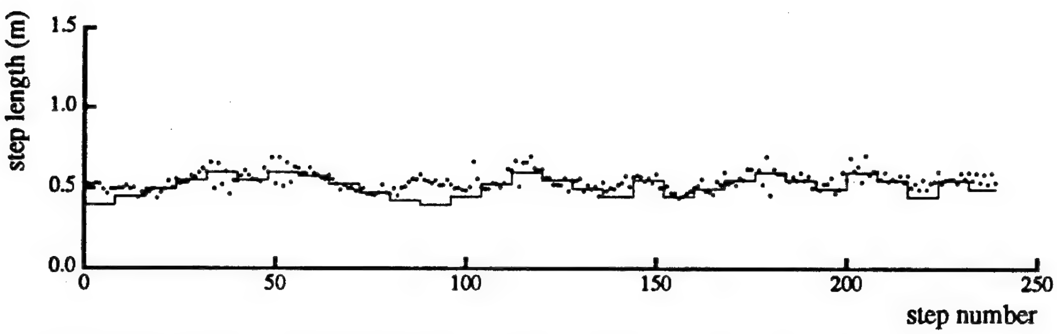

Figure 3–13: Stance duration is manipulated to adjust step length. The desired step length (solid line) was changed every eight steps, between 0.4 m and 0.6 m. The average error in step length was 0.03 m when the desired step length was within the range of step lengths possible with adjustments in stance duration. $\dot{x} = 1.1\,\mathrm{m/s}$, $T_f = 0.33\,\mathrm{s}$. (B88.265.7)

3.6 Comparison of the Methods

We compared the three methods for controlling step length using two metrics, accuracy and range. The accuracy is an error measure which reflects the difference between the desired and actual step lengths. The range is the difference between the minimum possible step length and the maximum possible step length. The accuracy of a method defines the size of a foothold the legged system could use successfully. If a legged system is to use a foothold that is 0.1 m long and 1.0 m away, then it must take a step of 1.05 m with an error of less than ±0.05 m. Otherwise it will not land on the foothold.

Good accuracy does not guarantee a successful method for controlling step length. A method for controlling step length must also be able to vary step length over a wide range. For example, if a system is running with a step length of 1 m but there is no good foothold 1 m ahead, then it must take a shorter or longer step to avoid stepping on the undesirable region of the terrain. If the undesirable region is large, the adjustment might have to be

Method	Controlled Variable minimum	Controlled Variable maximum	Controlled Variable nominal	Step Length minimum	Step Length maximum	Step Length range
forward speed	0.00 m/s	2.00 m/s	1.10 m/s	0.00 m	1.10 m	1.10 m
flight duration	0.14 s	0.50 s	0.40 s	0.32 m	0.72 m	0.40 m
stance duration	0.10 s	0.20 s	0.15 s	0.55 m	0.66 m	0.11 m

Table 3–1. The maximum and minimum values for each controlled variable and the maximum and minimum values for the step lengths produced by each method.

Method	Error in Adjusted Variable mean	Error in Adjusted Variable standard deviation	Error in Step Length mean	Error in Step Length standard deviation
forward speed	−0.091 m/s	0.111 m/s	0.05 m	0.07 m
flight duration	0.016 s	0.078 s	−0.04 m	0.09 m
stance duration	−0.001 s	0.014 s	−0.01 m	0.04 m

Table 3–2. The mean and standard deviation of the error in the adjusted variable and in the step length.

Method	# of trials	Error at Target mean	Error at Target standard deviation
no control of step length (theoretical)	none	0.000 m	0.320 m
forward speed	21	−0.004 m	0.023 m
flight duration	20	−0.051 m	0.046 m
forward speed + direct placement	25	0.001 m	0.004 m
flight duration + direct placement	25	0.000 m	0.005 m

Table 3–3. The mean and standard deviation of the error in foot placement on a target.

substantial.

Table 3–1 gives the minimum and maximum step lengths obtained with each method. Adjusting forward speed produced variations in step length that are more than twice the variations obtained by manipulating flight duration and more than ten times the variations of step length obtained by manipulating stance duration.

Table 3–2 shows the measured mean and standard deviation of the error in step length for each method. Only steps within the range of the method were used in the calculation of

the error. Manipulating stance duration provided the most accurate control, with a mean of -0.01 m and a standard deviation of 0.04 m for the error in step length. Manipulating forward speed and flight duration provide less accurate control of step length.

These data were obtained experimentally. The differences between the methods for controlling step length reflect characteristics of the mechanical apparatus and of the control algorithms.

3.7 Demonstrations

Using these methods to adjust the length of its steps, the planar biped can run on some simple forms of rough terrain. One such task is the problem of placing a particular foot on a target foothold. For these demonstrations the biped begins adjusting its step length about 5 m before it reaches the target foothold. Using the forward speed method for adjusting step length the mean and standard deviation of the error in foot placement is -0.004 m and 0.023 m. The mean and standard deviation of the error in foot placement for the flight duration method is -0.05 m and 0.046 m. With no control of step length, the error would be uniformly distributed with a range of plus or minus the step length (± 0.55 m) and the standard deviation of the error would be 0.32 m. When direct placement of the foot is combined with the forward speed or flight duration methods, foot placement error is essentially eliminated. Table 3–3 shows the mean and standard deviation of the error in the foot position at the target for each method.

The biped can reliably leap over obstacles by adjusting the length of its steps on the approach. The control system uses adjustments in step length to align the machine appropriately with the obstacle prior to the leap. When it reaches the desired takeoff point, the biped jumps as high as it can and shortens its legs to increase clearance. The machine has jumped over a rectangular obstacle 0.36 m high and 0.32 m long on fifteen consecutive attempts.

The biped can run up and down a flight of three stairs using these methods for controlling step length. Before the machine reaches the stairs, the step length is adjusted by manipulating the forward speed, so that the machine is in the correct position to begin the ascent. During the climb up the stairs, the control system uses the forward speed method to match the step lengths to the stair tread depth and it manipulates flight duration to account for stair riser heights.

3.8 Conclusions

A legged system must control the length of its steps if it is to use isolated footholds on rough terrain. The results described in this paper demonstrate that the step length of the planar biped, a dynamically stable legged robot, can be controlled. By adjusting the length of its steps, the planar biped has stepped on a target foothold, jumped over obstacles in its

path, and run up and down a flight of stairs.

Three variables affect the length of a step: the forward speed, duration of the flight phase, and the duration of the stance phase. Reducing any one of the three shortens the step; increasing each one lengthens the step. This observation motivated the development of three methods for controlling step length. Each method adjusts one of the variables to produce the desired step length and leaves the other two unchanged.

We compared the range and accuracy of each method for controlling step length. The forward speed method produced the widest range of step lengths. The range of step lengths produced by the flight duration method was about half that produced through by the forward speed method. The stance duration method produced step lengths with only a tenth the range produced by the forward speed method. When each method is used to follow a pattern of desired step lengths that falls entirely within its range, the stance duration method produced the highest accuracy.

Both range and accuracy are important for control of step length. The forward speed method appears to be the most practical of the three. It remains to factor out the degree to which these results are affected by the particular characteristics of the experimental apparatus, and other elements of the implementation.

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Chapter 4

How Fast Can a Legged Robot Run?

4.1 Abstract

Several parameters can limit the running speed of a legged system. Among them are the strength, length and stiffness of the legs, the range of joint motion, and the actuator force-velocity characteristics. We have explored how varying these parameters affects top running speed. We developed a dependency tree that suggests that a robot should have long, strong, stiff legs, and actuators with high peak velocity in order to run fast. We have also proposed three ways to improve the control of body attitude in high speed running: keeping hip motions symmetric, compensating for actuator characteristics, and accelerating the hip joint in anticipation of touchdown. In laboratory experiments a planar two-legged robot has reached a top speed of 5.9 m/s (13 mph).

4.2 Introduction

"How fast can it go?" Since time immemorial, people have staged races to compare the speed of people, animals, or vehicles. Speed excites people, often because it is the focus of a competition, and sometimes because of the danger or novelty of going fast. Speed is also an easily understood measure of performance. It summarizes the capabilities of a complex system with a single number that says something about an athlete's prowess, an animal's likelihood of survival, or a vehicle's utility.

Three things limit the speed of a vehicle: the power available to overcome drag, the ability of the structure to withstand loads, and the stability of the motion in the face of disturbances. As the system accelerates it encounters a limit of power, strength, or stability

that establishes its maximum speed. If the limitation is power, then at maximum speed the drag force cancels the thrust force, leaving no thrust to accelerate the system. If the limitation is strength, then at maximum speed the loading on some component equals its strength, and any increase in speed would cause it to break. If the limitation is stability, then at the maximum speed some equilibrating mechanisms is at its stability limit, and at any higher speed the system would tumble out of control. Most vehicles are designed so that their speed is limited by power, rather than by structure or stability, since a simple inability to accelerate is preferable to structural failure or loss of control.

How fast a legged system can run depends on its design, and on how it is controlled. The important parts of the design are the legs and the hips. To run fast, the legs should be long, strong, springy, and stiff, and the hips should be able to rotate rapidly and through a large angle. The control system must coordinate the actions of the legs and hips so as to regulate the momentum of the system in the horizontal, vertical, and rotational directions. The principles of symmetry, modeling, and anticipation help the control system to regulate rotation of the body.

Running speed is the product of step length and step frequency. The simple prescription for fast running is to take long steps, and to take them quickly. Step length depends on leg length and hip joint range of motion, while stepping rate depends on leg stiffness, leg strength, and hip rotation rate.

Leg length — Long legs allow long steps, so legs should be long for fast running. The distance that a legged system can move forward while its foot is on the ground is proportional to how long its legs are. Mass, moment of inertia, and strength all depend on leg length, and there is a limit to how big a leg can be and still be strong enough to support its own weight and the inertial forces required to move it.

Hip joint rotation — The distance that a legged system can travel forward during a bounce depends not only on the leg length, but also on the range of motion of the hip joint. The hip joint limits how far the leg can pivot during stance without disturbing the attitude of the body.

Hip rotation rate — Hip rotation rate can directly limit running speed. During stance, a legged system is like a polar manipulator. The foot remains fixed on the ground, and the leg length and hip angle determine the position of the body with respect to the foot. The hip position, plus the rate of extension of the leg and the rate of rotation of the hip joint determine the velocity of the body. The faster the hip joint can rotate, the faster the body can advance during stance.

Leg stiffness — A running system alternately bounces off of the ground and flies through the air. For the system to bounce, the legs must be springy. During each bounce against the ground, ground contact forces reverse the vertical momentum of the system. As the legs compress during stance, they build up force, and the vertical component of that force reverses the vertical momentum of the system. The stiffer the springs are, the faster the forces build up and the more quickly the system bounces.

Leg strength — The total impulse required to reverse the vertical momentum of the body is the integral of the contact force over the duration of the bounce. The legs must

be strong enough to transmit the ground reaction force to the body without breaking or buckling. The shorter the bounce, the larger the contact forces must be. Thus, the stronger the legs are, the faster the system can bounce off of the ground without damaging the legs.

Symmetric leg motions — By moving its two legs symmetrically, a bipedal minimizes how much its body attitude deviates from the nominal angel. If the hip joints are at the center of gravity of the body, then the only disturbances to the body attitude are caused by hip torques. Equal and opposite motions of the hip joints ensure that the hip torques cancel out, and thus do not disturb the body attitude.

Actuator velocity compensation — During stance, the hip of the stance leg is pushed forward by the body, causing the hip to rotate at a rate proportional to running speed. Velocity dependent torques in the hip joints should be compensated so that the body is not rotated forward with the leg.

Ground speed matching — When the leg touches down, an impulsive contact force brings the unsprung mass of the foot to rest. At high speeds, this impulse is not aligned with the axis of the stance leg, so it tends to rotate the leg. The impulse happens very quickly, faster than the hip joint can respond, so some torque is transmitted to the body, which also begins to rotate. If the control system anticipates the touchdown, and accelerates the hip joint before impact, then the impulse is aligned with the axis of the leg. In this case the hip joint does not transmit any torque to the body.

Taken together, symmetric leg motions, actuator velocity compensation, and ground speed matching substantially reduce body attitude disturbances associated with running fast.

In the following section we review relevant previous studies. Then we develop a dependency tree that expresses the speed of a running system in terms of its physical parameters. Finally, we present laboratory experiments suggested by the analysis.

4.3 Background

The study of running is interdisciplinary. Some areas of research that provide results helpful in understanding running speed are:

- Studies of running animals
- Creation of artificial legged systems (robots)
- Investigation of the performance of vehicles in general

Biologists have studied innumerable aspects of running animals, including their structure, the motions and forces that occur during running, and the energy consumed for different speeds and gaits. Robots and vehicles that travel on legs provide a way to study walking and running in simple, easily instrumented systems, without the complexity inherent to biological systems. Research into the performance of boats, aircraft, and land vehicles is relevant to studying running speed, because the task of locomotion and the

physical principles of support, balance, and progress are common to all vehicles.

4.3.1 Biomechanical Research

In the 1870s, photographer Eadweard Muybridge studied racehorses. He took a series of photographs in rapid succession, one of which showed that a trotting horse had all four feet in the air at once (Muybridge, 1878). He went on to take serial photographs of hundreds of moving animals (Muybridge, 1887). The camera froze the motion so that it could be studied, and Muybridge used the pictures to determine the pattern of footfalls that characterized different gaits. He also attached series of photographs around the inside of a cylinder, with slits between the pictures. By spinning the cylinder and looking through the slits, the viewer saw each picture in sequence, and the pictures appeared to move. This device, called a zoetrope, was a precursor of the motion picture projector. Muybridge's study anticipated modern cinematic studies, and his pictures continue to be a valuable reference for anyone studying legged locomotion.

Scaling

Biologists have proposed various similarity models to describe how the shape, structure, and motions of an animal depend on its size. Similarity models either describe measurements gathered from animals that are similar in arrangement but vary in size, or they describe the way that such measurements should vary in order to keep some quantity the same at all sizes.

Similarity models apply to all mechanisms, not just biological ones. A mechanism design only works over a limited range of sizes. Typically, at a very small size the ratio of viscous forces to inertial forces decreases and the resultant damping prevents the mechanism from operating. At a very large size, gravitational forces become dominant and exceed the strength of the materials.

Hill (1950) concluded that speed is independent of size for animals of similar design. An animal makes movements that are proportional to its size, but at a frequency that is inversely proportional to its size. For geometrically similar animals, the differences cancel out, so top speed is the same regardless of size. Hill introduced the idea of physiological time, saying that animals live on a time scale proportional to body size. Thus large animals live longer than small animals, their hearts beat more slowly, and they take more time for each running step. Hill also noted that for large animals, a greater portion of skeletal and muscular strength is required to support the animal's weight than for small animals.

McMahon (1975) compared and discussed three scaling laws: geometric similarity, elastic similarity, and static stress similarity. Geometrically similarity preserves shape across scale, as all linear dimensions change with the same scale factor. Elastic similarity preserves resistance to column buckling. For columns of different size to have the same safety factor against buckling, long columns must be relatively thicker than short columns. The scale factor for the diameter of elastically similar columns is 3/2 the scale factor for length. Static

Comparison of Three Similarity Models

	geometric similarity	elastic similarity	static stress similarity
length, ℓ	$\ell \propto W^{1/3}$	$\ell \propto W^{1/4}$	$\ell \propto W^{1/5}$
diameter, d	$d \propto W^{1/3}$	$d \propto W^{3/8}$	$d \propto W^{2/5}$
surface area, S	$S \propto W^{2/3}$	$S \propto W^{5/8}$	$S \propto W^{3/5}$
cross sectional area, A	$s \propto W^{2/3}$	$A \propto W^{3/4}$	$A \propto W^{4/5}$
natural frequency, ω	$\omega \propto W^{-1/3}$	$\omega \propto W^{-1/8}$	$\omega \propto W^{0}$
speed, V	$V \propto W^{0}$	$V \propto W^{1/8}$	$V \propto W^{2/5}$

Table 4–1. Three similarity principles predict different variations in shape and speed as a function of body weight, W. The table is adapted from a paper by McMahon (1985), who cites examples of animal measurements that match the elastic similarity model.

stress similarity preserves resistance to bending failure in simply supported beams bearing their own weight. In this case, the scale factor for diameter is twice the scale factor for length.

McMahon presented a variety of evidence that the design of animals is in accordance with elastic similarity. In particular, the running speed of animals is proportional to body weight raised to the 1/4 power, as predicted by elastic similarity. Geometric similarity predicts an exponent of zero, and static stress similarity predicts an exponent of 2/5. Table 4–1 includes a few predictions of the three similarity models.

Alexander (1988) found that animals weighing over 20 kg scale according to elastic similarity, while the dimensions of smaller animals obey geometric similarity.

Alexander also describes an extension of geometric similarity called dynamic similarity. For geometric similarity, animals of different sizes undergo a uniform scaling of linear dimensions. For dynamic similarity, the scaling of linear dimensions is accompanied by uniform scaling of time and force. Thus, dynamic similarity specifies not only the change in shape of animals at different sizes, but also changes in their motions. Alexander proposes that the scale factor for forces be the cube of the scale factor for length, and that the scale factor for time be the square root of the scale factor for length. These scale factors work for motions characterized by gravitational and inertial forces. However, Alexander points out that if both elastic and gravitational forces are important it is impossible to maintain strict dynamic similarity.

Energetics

The power required for running increases with speed. The environment opposes motion with drag forces, dissipating power equal tot he product of the forward speed and the drag force. Drag forces remain constant or increase with running speed. For example, the gravitational drag caused by climbing a hill is independent of speed, while aerodynamic drag increases as the square of running speed.

Several investigators have measured the oxygen consumption of running animals. Consumed oxygen produces metabolic energy at a rate of 1 ml O_2 = 20.1 J. The rate of oxygen consumption indicates the metabolic power produced by the animal, some of which is used to overcome the resistance of the environment, some of which is dissipated in muscle inefficiency, and some of which is used to maintain the animal's basal metabolism.

The rate of metabolic energy consumption increases with running speed. Taylor, Heglund, and Maloiy (1982) report that the energy consumed during running is:

$$\dot{E}_{\mathrm{metab}}/M_b = 10.7 M_b^{-0.316} v_g + 6.03 M_b^{-0.303} \tag{4.1}$$

where E_{metab} is the metabolic energy consumed in watts, M_b is the body mass in kg, and v_g is the running speed in m/s. The equation is based on measurements from 60 species of animals, ranging in size from 0.0072 kg pygmy mice to 254 kg zebu cattle. It indicates that energy consumption increases linearly with running speed, and that the rate of increase is smaller for large animals than for small animals.

Dawson and Taylor (1973) studied the energetic cost of locomotion in kangaroos. They found that over a range of speeds from 2 m/s to 6 m/s, hopping frequency and energy consumption remain nearly constant, and the stride length increases in proportion to the speed. This is in contrast to the linear increase in energy consumption with speed indicated by equation 4.1. Dawson and Taylor attributed the constant consumption of energy with increasing speed to increased storage and recovery of elastic energy, particularly in the kangaroo's large Achilles tendon.

Alexander and Vernon (1975) measured the ground forces exerted by hopping kangaroos using force plates, and combined the measurements with film records of the motion to determine the fluctuations of energy during hopping. They calculated that elastic storage of energy in the kangaroo's Achilles tendon reduced the energetic cost of hopping by 40%. Alexander and Vernon also noted that the kangaroo's tail rotates in the opposite direction from the legs, in a way that reduces the angular motions of the body. Large sheets of elastic tendon along the tail contribute to its oscillation.

McMahon and Greene (1978, 1979) considered not only the compliance of a runner's legs, but the compliance of the ground as well. Their model predicted that top running speed would be slightly greater on a compliant track than on a rigid surface. The model predicted that the fastest running would be on a track four times as stiff as the runner's legs. McMahon and Greene built such a track at Harvard University, and observed the predicted 2% increase in running speed, along with a decrease in injuries. This study points out the importance of the environment in determining the top speed of a running system.

Hoyt and Taylor (1985) studied the energetics of horses walking, trotting and galloping. For each gait they found one speed that provided the best energy efficiency, and if allowed to move freely, a horse always chose a speed and gait that corresponded to a local maximum in efficiency.

4.3.2 Robotic research

There are very few artificial systems that walk or run. Walking and running require dynamic stability, meaning that they move continuously in order to keep the average point of support beneath the center of gravity.

Most legged robots and legged vehicles crawl. Crawling requires static stability, meaning that there is always a polygon of support, and that the projection of the center of gravity onto the ground always falls within the polygon of support. A polygon of support requires at least three feet on the ground at all times, unless the feet are large. A four legged vehicle can creep along, using three legs to provide a tripod of support while it repositions the fourth foot. A six legged vehicle often uses an alternating tripod gait, in which three of the legs provide support while the other three move forward for the next step.

Table 4–2 lists the number of legs, leg length, and speed of several legged robots. Keep in mind that none of these systems was designed for speed. Each was designed to study some aspect of the control of locomotion.

Two similar robots, the OSU Hexapod (Jaswa, 1978) and the integrated walking robot (Gurfinkle et al., 1981) were six-legged robots used for experiments in basic control, gait selection, and terrain adaptation. The ODEX I (Russel, 1983) is a six-legged robot that uses a pantograph leg design. The legs have a large range of motion, allowing ODEX to adopt a wide base or a narrow profile as required for the task at hand. The SSA Hexapod (Sutherland and Ullner, 1984) was a six legged robot that combined electronic and hydraulic computing to simplify control. The Adaptive Suspension Vehicle (ASV) (Waldron et al. 1984) carries a driver on board. The ASV also uses a pantograph leg design. The pantograph legs allow the body to move forward at constant altitude without requiring any of the leg joint actuators to do negative work. The ASV legs also have a large range of motion, which allow it to traverse substantial obstacles. The ASV is the largest and fastest of these six-legged systems. It has a leg length of about 1.9 m and a design speed of 3.5 m/s. The demonstrated speed is about 2 m/s.

Hirose (1984) developed the PV II (Perambulating Vehicle Mark II), a four-legged, statically stable robot. It has foot sensors and reflexes that allow it to sense and climb stairs. A dramatically different four-legged robot is the quadruped developed by Raibert (1986), which trots, paces, and bounds in the laboratory.

Kato (1983) developed the WL-10RD, a two-legged walking robot. For most of each step cycle, it was statically stable, with the center of mass balanced above a large foot. Once during each cycle, it fell forward, transferring support from one foot to the other. Miura and Shimoyama (1984) built Biper-3, a two-legged machine that walked in a way reminiscent of a person on stilts. It had three actuated joints, one to swing each leg in or out sideways, and a third to scissor the legs forward and backward. None of these machines traveled very fast.

Huang and Waldron (1987) derived the relationship between weight and maximum speed for a hexapod vehicle crawling with a particular gait. By assuming that the distribution of forces among the support legs was a linear function of the forward and lateral

Legged Machines Robots and Vehicles

	Leg Length m	Speed m/s	Speed mph
6 legs			
OSU Hexapod (McGhee)	0.8	0.3	0.7
USSR Hexapod (Gurfinkle)	0.35	0.1	0.2
SSA Hexapod (Sutherland)	1.0	.14	0.3
ODEX (Odetics)			
ASV (Waldron)	1.90	2.2	5
4 legs			
PV II (Hirose)	0.87	0.5	1.1
Quadruped (Raibert)	0.66	3	6.7
2 legs			
WL10-RD (Kato)	0.96	.23	.51
Biper-3 (Miura)	0.20	0.02	0.04
MEG-2 (Funabashi)	0.48	0.5	1.1
Kenkyaku (Furusho)	0.72	0.8	1.8
Planar Biped	0.80	5.9	13.1

Table 4–2. These are a few of the machines and vehicles that have been used to study the control of legged locomotion. None of the machines was explicitly designed for high speed. All of the leg lengths and speeds are approximate.

position of the legs, and requiring the vehicle to remain in static equilibrium, they determined the proportion of the vehicle's weight on each leg as a function of speed. By limiting the force on the most heavily loaded leg to the maximum safe load they computed the tradeoff between speed and payload.

4.3.3 Vehicular Research

Gabrielli and von Kármán (1950) studied the cost of locomotion at different speeds. They gathered data on the gross weight, installed power, and maximum speed of many land vehicles, ships, boats, aircraft and animals. For each vehicle they computed the *specific resistance*, which is the ratio of power to the product of weight and velocity: $\epsilon = P/WV$. This nondimensional quantity is a measure of the energetic cost of locomotion. Gabrielli and von Kármán's data indicate that any particular means of locomotion is only energy efficient over a narrow range of speeds, and that in general small, fast vehicles are less

efficient than large, slow vehicles. For example, a merchant ship has a specific resistance of about 0.003 at a speed of 6 m/s, while a jet fighter plane has a specific resistance of 0.3 at a speed of 300 m/s.

The graphs of specific resistance as a function of speed show a limiting line, a minimum specific resistance that increases with speed. This line represents an efficiency limit imposed by aerodynamic or hydrodynamic drag. Single vehicles of sufficient size should have specific resistances below the limit, because specific resistance decreases with vehicle size. Railroad trains achieve a greater energetic efficiency because the aerodynamic drag is smaller for the train than for the individual cars, and because the power is supplied by a few large, efficient engines. Gabrielli and von Kármán point out that the size of the vehicles that can be built is limited by the strength to weight ratio of the available construction materials.

4.4 The Dependency Tree

One way to impose structure on the relationship between the physical parameters of a mechanism and how fast it can run is to build a dependency tree, as shown in figure 4–1. The top of the tree is running speed. The branches are formed by expressing running speed as the ratio of step length to step period, and successively refining those quantities to simpler characteristics of the running motion. The leaf nodes are parameters of the links, joints, and actuators. Body attitude is an exception; it is determined by how well the control system corrects disturbances.

The equations in figure 4–1 embody several assumptions, which are explicitly stated in the paragraphs below. The resulting analysis accurately represents the kinematics of the mechanism, but it uses simplified dynamics, and says nothing about energetics. It leads to tractable expressions for running speed that predict, at least qualitatively, how parameter variations affect speed.

Speed

The first row of the dependency tree in figure 4–1 is the definition of running speed (V), which is the ratio of the forward progress on each step (S) to the time required to complete that step (T):

$$V = S/T. \tag{4.2}$$

To increase its speed, a running system must take longer steps, more frequent steps, or both. In bipedal running, stance and flight proceed in strict alternation, and a step consists of exactly one stance and one flight. Figure 4–3 shows that the distance traveled during one step is the sum of the distance traveled during stance (S_s) and the distance traveled during flight (S_f):

$$S = S_s + S_f. \tag{4.3}$$

Likewise, the time required for a step is the sum of the stance duration (T_s) and the flight

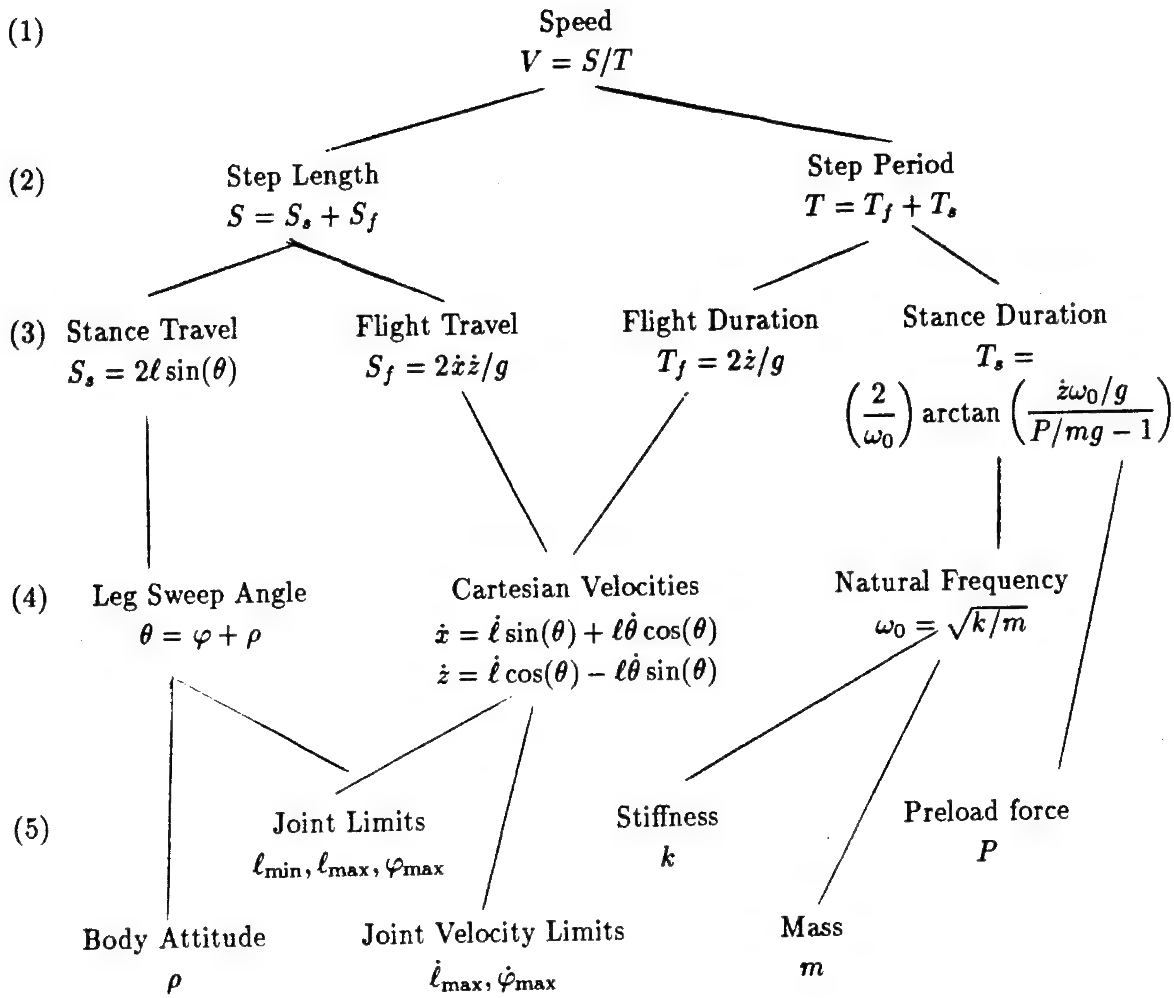


Figure 4–1: A tree indicates some of the ways in which the running speed of a legged system depends upon parameters of the mechanism. The top of the tree represents speed, a measure of performance. The intermediate rows represent characteristics of the running motion, and the bottom row represents the parameters of the mechanism. The state variables of the system are the leg length, the rate of leg extension, the leg angle, and the rate of leg rotation. The operating point is described by the state at the moment of liftoff: ℓ, $\dot{\ell}$, θ, and $\dot{\theta}$. The horizontal and vertical components of liftoff velocity are $\dot{x}$ and $\dot{z}$. If the steps are symmetric and the pitch angle of the body is uniformly zero, then the operating point completely characterizes the motion. Steps are symmetric if the horizontal velocity and the leg length are the same at touchdown as at liftoff, and the vertical velocity and the leg angle change signs from touchdown to liftoff. The coordinate system is shown by figure 4–2.

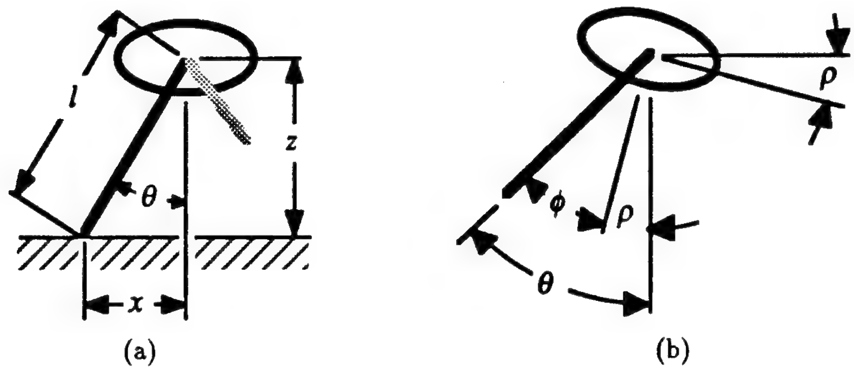


Figure 4–2: (a) The horizontal and vertical distances from the foot to the hip are determined by the leg length and leg angle: $x = \ell \sin(\theta)$, $z = \ell \cos(\theta)$. During stance the foot is motionless, so the derivatives of the hip coordinates give the horizontal and vertical velocity with respect to the ground: $\dot{x} = \dot{\ell} \sin(\theta) + \ell\dot{\theta}\cos(\theta)$, $\dot{z} = \dot{\ell}\cos(\theta) - \ell\dot{\theta}\sin(\theta)$. (b) The leg angle is the sum of the leg angle with respect to the body, and angle of the body with respect to the ground: $\theta = \varphi + \rho$. The three angles, θ, φ, and ρ are measured clockwise from the nominal position, in which the leg is vertical and the body horizontal.

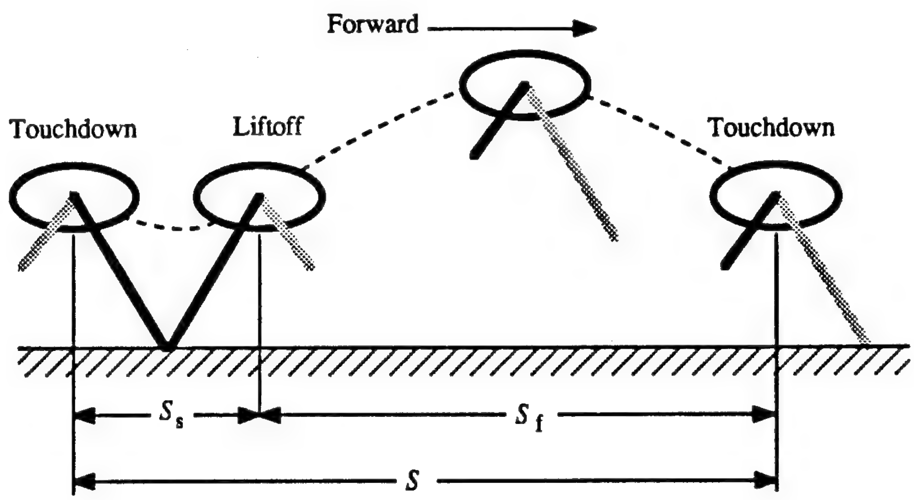


Figure 4–3: Step length (S) is the sum of the forward progress of the hip during stance (S_s) and the forward progress of the hip during flight (S_f).

duration (T_f):

$$T = T_s + T_f. \tag{4.4}$$

These definitions make up the second row of the tree.

In the third row of the tree, the dynamics of the system come into play. In the stance phase, the system resembles both a mass bouncing on a spring and an inverted pendulum

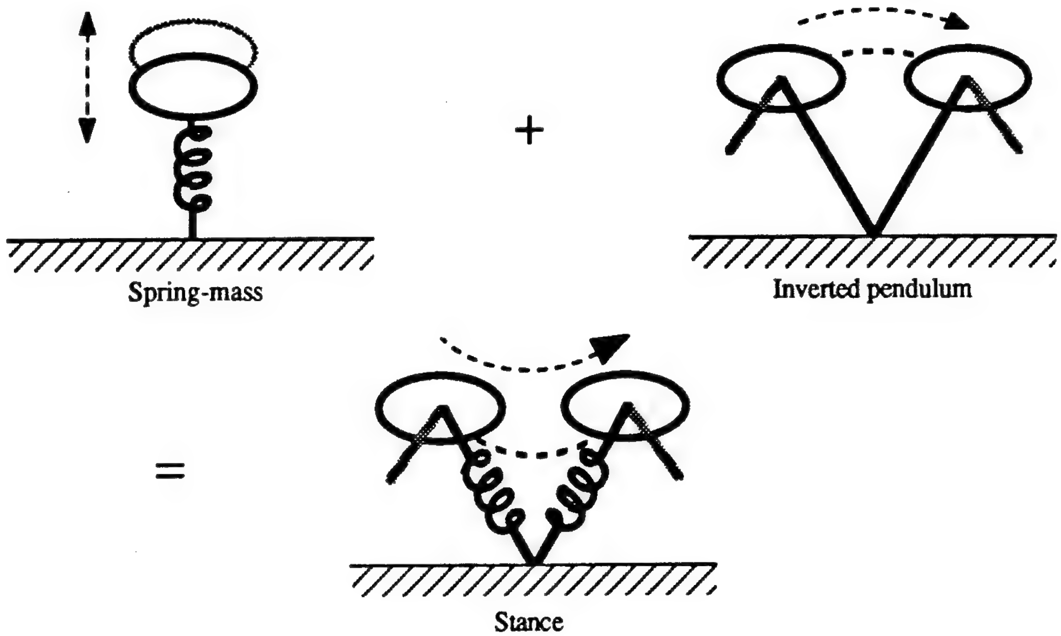


Figure 4–4: The motion of a running system during stance results from the interaction of two simpler motions. The vertical motion is predominantly the bouncing motion of a spring-mass oscillator. The forward travel results from the tipping motion of an inverted pendulum that moves first toward and then away from the unstable equilibrium point. Forward speed decreases during the first half of stance, because some of the horizontal kinetic energy is temporarily stored in the leg spring. During the second half of stance, the spring releases energy and the system speeds back up.

pivoting over its fulcrum, as shown in figure 4–4. The dynamics are much simpler during flight, when the system approximates a rigid ballistic projectile rising and falling under the influence of gravity.

Stance

The leftmost and rightmost nodes of row three describe the stance duration and the stance travel. The stance travel is the forward progress of the body during stance, and is a function of the leg lengths and leg angles at the beginning and end of the stance phase, as shown in figure 4–5. For symmetric steps, the leg length and leg rotation rate are the same at touchdown and at liftoff, while the leg extension rate and the leg angle have opposite signs at touchdown and at liftoff. Under the assumption of symmetry, the distance traveled during stance is:

$$S_s = 2\ell \sin(\theta), \tag{4.5}$$

where θ and ℓ represent the leg angle and leg length at liftoff.

The motion during stance is described by a pair of coupled second-order non-linear differential equations. The stance duration can be computed by integrating these equations

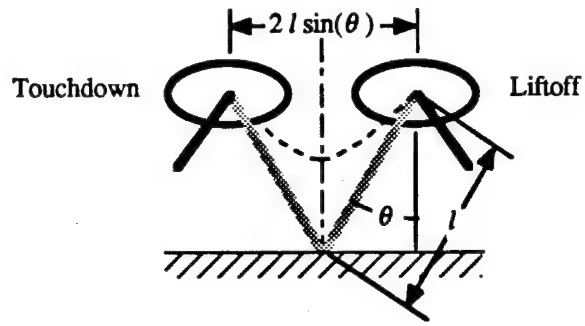


Figure 4–5: The distance traveled during stance is a function of the leg length and hip angle at the beginning and end of stance. If the motion is symmetric, so that the state at liftoff is a mirror image of the state at touchdown, the distance traveled is $2\ell\sin(\theta)$. The system in the figure is moving from left to right, touching down with the foot in front of the hip and lifting off with the foot behind the hip.

forward in time from the moment of touchdown until the moment of liftoff. McMahon and Cheng (1989) have numerically solved the equations for a number of cases in order to determine the linear leg spring stiffness required to run at a given constant speed with a given leg angle at touchdown and liftoff.

We know of no closed form expression for the stance duration as a function of the mechanism parameters and the state of the system at touchdown. In order to proceed with the analysis, we pretend that the horizontal and vertical motion of the system are decoupled, and that the stance duration is determined only by the vertical motion.

For the simple case of a mass bouncing on a linear spring, McMahon (1986) showed that the time required to rebound from the ground depends on a parameter he called the Groucho number. The Groucho number is a dimensionless group composed of the touchdown velocity, the natural frequency of the spring-mass system. and the acceleration of gravity:

$$
\begin{aligned}
T_s &= (2\pi/\omega_0)\,[1 - (1/\pi)\arctan(N_G)] \qquad (4.6)\\
N_G &= \dot{z}\omega_0/g\\
\omega_0 &= \sqrt{k/m}.
\end{aligned}
$$

where

- N_G is the Groucho Number,
- ω_0 is the natural frequency of the spring-mass system,
- $\dot{z}$ is the vertical velocity at liftoff,
- g is the acceleration of gravity
- k is the leg stiffness, and
- m is the body mass.

Equation 4.6 expresses the stance duration as the product of two factors. The first factor is the time required for a full cycle of oscillation, which depends on the natural frequency. The

second factor is the proportion of a full cycle required to rebound from the ground, which depends on the Groucho number. If the mass has no downward velocity at touchdown, the system spends a full cycle on the ground. At the opposite extreme, with infinite downward velocity, the rebound takes only a half cycle of oscillation.

Equation 4.6 is correct if the leg force is zero at touchdown and liftoff. If a mechanical stop prevents the leg spring from extending to its full length, then the force at touchdown is the preload force that results from keeping the spring compressed. The leg force throughout stance is augmented by the preload force, and the stance is shorter than it would be without the extra force. With a preload force, the Groucho number should be modified:

$$\begin{aligned} N'_G &= \frac{N_G}{P/mg - 1} \\ &= \frac{\dot{z}\omega_0/g}{P/mg - 1} \end{aligned} \tag{4.7}$$

where N'_G is the modified Groucho number and P is the preload force. With the modified Groucho number, the formula for stance duration is:

$$T_s = \begin{cases} (2/\omega_0)\left[\pi - \arctan\left(N'_G\right)\right], & \text{if } N'_G > 0 \quad (P < mg); \\ \dfrac{\pi}{\omega_0}, & \text{if } N'_G = \infty \quad (P = mg); \\ (2/\omega_0)\arctan\left(N'_G\right), & \text{if } N_G < 0 \quad (P > mg). \end{cases} \tag{4.8}$$

The entry for stance duration in figure 4–1 is the third case of equation 4.8, where the preload force exceeds the weight of the system.

Equation 4.8 gives the stance duration of a system bouncing in place on a linear spring. Increasing the leg stiffness, the preload force, or the vertical velocity shortens the stance duration, whereas increasing the mass, or the acceleration of gravity lengthens the stance duration. The behavior is qualitatively similar for the more complex case of a nonlinear spring and the leg pivoting about the foot. In the nonlinear case, equation 4.8 can be used to predict stance duration by assuming, computing, or measuring a value for the natural frequency. The natural frequency depends on the effective vertical stiffness, which depends on the sweep angle and impact velocity as well as on the stiffness of the leg.

Flight

The two nodes in the middle of row three of figure 4–1 describe the flight travel and the flight duration. The formulas give the time of flight and forward progress of a rigid body that has an initial velocity $(\dot{x}, \dot{z})$ and is accelerated only by gravity (g). The formulas thus ignore changes in the location of the center of gravity due to the motions of the legs, and accelerations due to aerodynamic drag. The rigid body assumption leads to simple expressions for the flight duration and the flight travel:

$$T_f = 2\dot{z}/g \tag{4.9}$$

$$S_f = 2\dot{z}\dot{x}/g. \tag{4.10}$$

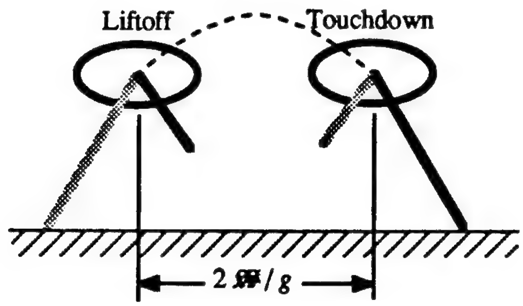


Figure 4–6: The distance traveled during flight is determined by the liftoff velocity $(\dot{x}, \dot{z})$. For symmetric steps, in which the height of the body above the ground is the same at touchdown as at liftoff, the duration of flight is $2\dot{z}_{lo}/g$. The forward progress during flight is $2\dot{z}\dot{x}/g$. The system in the figure is moving from left to right, lifting off from the foot behind the hip and landing on the foot in front of the hip.

The third row of figure 4–1 divides running speed into four quantities: stance travel, stance duration, flight travel, and flight duration. These four quantities depend on the quantities in row four, which are functions of the operating point, and on the parameters in row five, which characterize the mechanism. The behavior in stance depends on the mass of the system, the stiffness and preload of the leg spring, the length and angle of the leg at liftoff, and the regulation of body attitude. The behavior in flight depends on the joint positions and velocities at liftoff. The following sections discuss the dependencies on operating point and mechanism parameters in more detail.

Stance Travel

The system travels a distance $S_s = 2\ell \sin(\theta)$ during stance. The motion during stance depends on the leg length and leg angle at touchdown, which are chosen by the control system to make the motion during stance symmetric and thus maintain a constant forward speed. The higher the speed, the farther forward the foot must be ahead of the hip at touchdown. The design of the leg limits the leg length, and the design of the hip joint limits the angle of the leg with respect to the body.

The leg angle is limited by the angle of the leg with respect to the body, and by the angle of the body with respect to the ground. We call the angle of the body with respect to the ground the pitch of the body. With the body in its nominal orientation, the range of leg angles permitted by the hip joint is symmetric about vertical, allowing the leg to swing equally far forward and backward. Deviations of the body from its nominal orientation reduce either the distance that the leg can reach forward for touchdown or the distance that it can reach back before liftoff. Either case reduces top speed by reducing the travel that can be achieved during stance.

If the body rotates away from its nominal pitch angle, the travel of the foot with respect to the hip is asymmetric. If the body is pitched forward, the distance that the foot can

reach ahead at touchdown is reduced. During flight, the control system positions the foot in front of the hip, by a distance that is proportional to running speed. Reducing the foot travel ahead of the hip reduces the maximum stable running speed, regardless of how much foot travel is gained behind the hip. If the body rotates backward, the foot can reach farther ahead of the hip for touchdown. However, the hip joint then reaches its limit of rearward travel before the end of stance, abruptly pitching the body forward.

Stance Duration

The duration of stance is $T_s = (2/\omega_0)\arctan\left(\frac{\dot{z}\omega_0/g}{P/mg-1}\right)$. The expression is based on the assumption that the stance duration is determined by the vertical motion of the system independent of the horizontal motion. Other plausible expressions for the stance duration might be derived from the strength of the leg, from the decrease in forward speed during stance, and from the velocity and acceleration limitations of the hip joints. For figure 4–1, we chose the representation based on stiffness, because leg stiffness *determines* stance duration, while leg strength and hip joint properties might *limit* stance duration. The stance duration must be long enough that the forces do not break the leg, and that the hip joint has time to move from the touchdown angle. The stance duration must be long enough, or the speed slow enough, that the hip joint does not exceed its maximum angle of rotation.

Flight Travel and Flight Duration

During flight the center of gravity of the system moves along a parabolic trajectory determined by the velocity at liftoff and the acceleration of gravity. The liftoff velocity depends on the leg length, leg extension rate, leg angle and leg rotation rate. It has magnitude $\sqrt{\ell^2\dot{\theta}^2 + \dot{\ell}^2}$ and direction $\arctan(\dot{\ell}/\ell\dot{\theta}) - \theta$. The magnitude is independent of the leg angle. The horizontal and vertical components are:

$$\dot{x} = \dot{\ell}\sin(\theta) + \ell\dot{\theta}\cos(\theta) \tag{4.11}$$
$$\dot{z} = \dot{\ell}\cos(\theta) - \ell\dot{\theta}\sin(\theta). \tag{4.12}$$

Speed Equations

Combining the formulas in figure 4–1 yields a single equation that expresses running speed as a function of operating point and mechanism parameters. The top row of the tree defines running speed as $V = S/T$. Breaking up the step length and step period expands the definition to:

$$V = \frac{S_f + S_s}{T_f + T_s}. \tag{4.13}$$

Incorporating the definitions of stance travel, flight travel, flight duration, and stance du-

ration gives:

$$V = \frac{\dot{x}\dot{z} + g\ell \sin\theta}{\dot{z} + (g/\omega_0)\arctan\left(\dfrac{\dot{z}\omega_0/g}{P/mg-1}\right)}. \tag{4.14}$$

Finally, replacing the Cartesian components of liftoff velocity according to equations 4.11 and 4.12, gives an equation for running speed in terms of the state variables $(\ell, \dot{\ell}, \theta, \dot{\theta})$ at liftoff and the parameters (ω_0, P, m, g):

$$V(\ell, \dot{\ell}, \theta, \dot{\theta}, \omega_0, P, m, g) \tag{4.15}$$

$$= \frac{(\dot{\ell}^2 - \ell^2\dot{\theta}^2)\sin(2\theta) + 2\ell\dot{\ell}\dot{\theta}\cos(2\theta) + 2g\ell\sin(\theta)}{2\dot{\ell}\cos(\theta) - 2\ell\dot{\theta}\sin(\theta) + (2g/\omega_0)\arctan\left\{\left(\dfrac{\omega_0/g}{P/mg-1}\right)\left[\dot{\ell}\cos(\theta) - \ell\dot{\theta}\sin(\theta)\right]\right\}}.$$

The extent of the state space is determined by mechanism parameters that describe maximum excursions and velocities of the joints. The construction of the leg establishes minimum and maximum leg lengths, and the rate of change of leg length is less than some maximum. If the pitch angle is always zero, then the leg angle and leg rotation rate are limited by the maximum excursion and velocity of the hip joint. At liftoff the leg angle, leg rotation rate, and leg extension rate are all positive:

$$\begin{aligned} \ell_{\text{min}} &\le \ell \le \ell_{\text{max}} \\ 0 &\le \dot{\ell} \le \dot{\ell}_{\text{max}} \\ 0 &\le \theta \le \varphi_{\text{max}} < \pi/2 \\ 0 &\le \dot{\theta} \le \dot{\varphi}_{\text{max}}. \end{aligned} \tag{4.16}$$

The operating point, which is the state at liftoff, lies in this restricted region of the state space.

Summary of the Dependency Tree

The top of the dependency tree shown in figure 4–1 is a performance measure, running speed. The intermediate rows are characteristics of the running motion:

- step rate
- step length
- flight duration
- stance duration
- stance travel
- flight travel
- leg sweep angle
- horizontal and vertical velocity at liftoff
- natural frequency

At the bottom of the tree are the parameters of the physical mechanism that determine running speed:

- body attitude
- leg length
- hip position
- leg extension rate
- hip rotation rate
- leg spring preload force
- leg stiffness
- mass

The formulas in the dependency tree are based on the following assumptions:

- There is no air drag, so the speed during flight is constant.
- The center of gravity is fixed with respect to the body, so the mechanism moves like a rigid body during flight.
- The motion during stance is symmetric:

$$\ell_{td} = \ell_{lo} \qquad \theta_{td} = -\theta_{lo} \qquad x_{td} = -x_{lo} \qquad z_{td} = z_{lo}$$
$$\dot{\ell}_{td} = -\dot{\ell}_{lo} \qquad \dot{\theta}_{td} = \dot{\theta}_{lo} \qquad \dot{x}_{td} = \dot{x}_{lo} \qquad \dot{z}_{td} = -\dot{z}_{lo}$$

This structure provides a framework for studying how running speed depends on the operating point and on the physical mechanism parameters.

The dependency tree provides a model of running speed. Although it incorporates several simplifying assumptions, it provides more intuition about how running speed depends on mechanism parameters than more complex models would. In particular, dynamic simulations would predict the results of experiments, but they have too many parameters to offer much insight to the problem. The dependency tree models running with a small number of parameters, and shows qualitatively how each affects speed.

4.5 Experiments

We have experimented with the planar biped to study how the design and control of a legged system affect its top running speed. The planar biped runs faster with long legs than with short legs, and faster with stiff leg springs than with soft leg springs. Experimenting with the biped has made it clear that there are speed dependent disturbances to body attitude, and that fast running requires that the control system reject or correct those disturbances. The biped's power dissipation increases with running speed, but the increase is small compared with the power required just to run in place.

4.5.1 How Fast Running Differs from Slow Running

The legs and body oscillate during running, and the frequency and amplitude of the oscillations are larger for fast running than for slow running. Figures 4–7, 4–8, and 4–9 show the motions of the legs and body as the biped ran forward at a constant speed of 1, 3, and 5 m/s, respectively. Table 4–3 lists the properties of a typical step at each speed. Compared to running slowly, running fast was characterized by longer and more frequent steps, higher frequency oscillations of leg length, leg angle, and body attitude, smaller and more frequent vertical oscillations of the body, and larger angular motions of the legs.

The biped's higher stepping frequency when it ran fast resulted from shorter flight duration, and to a lesser extent from shorter stance duration. The flight duration was lower because when the biped ran fast it took shallower bounces, and thus took less time to fall back to the ground. The stance duration was shorter because the shallower bounces meant that there was less vertical momentum to reverse during stance, and because the larger sweep angle increased the effective stiffness of the leg.

The thrust algorithm affected the way that hopping height varied with running speed. In these experiments, the leg actuator delivered as much energy as possible during stance by extending as fast as it could. The energy added by thrusting was dissipated by the vertical and horizontal motions of the system. At low running speeds, the horizontal motion dissipated very little energy, so on each step the system bounced high enough that the vertical motion dissipated most of the energy. At high running speeds, the horizontal motion dissipated more energy, and the system did not bounce as high. A thrust algorithm that kept the hopping height constant would have added more energy at higher running speeds, eliminating the change in flight duration as speed increased.

The stance duration was shorter for fast running because of the smaller vertical motion and the larger leg angles. The reduced vertical oscillation had lower touchdown and liftoff velocities, requiring a smaller impulse to reverse the vertical momentum during stance. The larger leg angle at touchdown meant that the leg spring was compressed more when the leg was vertical, and the larger leg force more quickly reversed the vertical momentum of the system.

The step length was much longer for running at 3 m/s than it was for running at 1 m/s, but only slightly longer at 5 m/s than at 3 m/s. Twice as much of the increase from 1 m/s to 3 m/s was due to the increase in step length as was due to the decrease in step period, but from 3 m/s to 5 m/s, ten times as much of the increased speed was due to the shorter step period.

Both the stance travel and the flight travel were larger at 3 m/s than at 1 m/s. At 5 m/s, the stance travel had increased a little more, but the flight travel had not. The longer stance travel means that the legs rotated through a larger angle when running fast than when running slowly. Flight travel increased between 1 m/s and 3 m/s because the increase in horizontal velocity during flight more than compensated for the reduced flight duration at the higher speed. From 3 m/s to 5 m/s the greater horizontal speed canceled the shorter flight duration, so the flight travel was the same at both speeds.

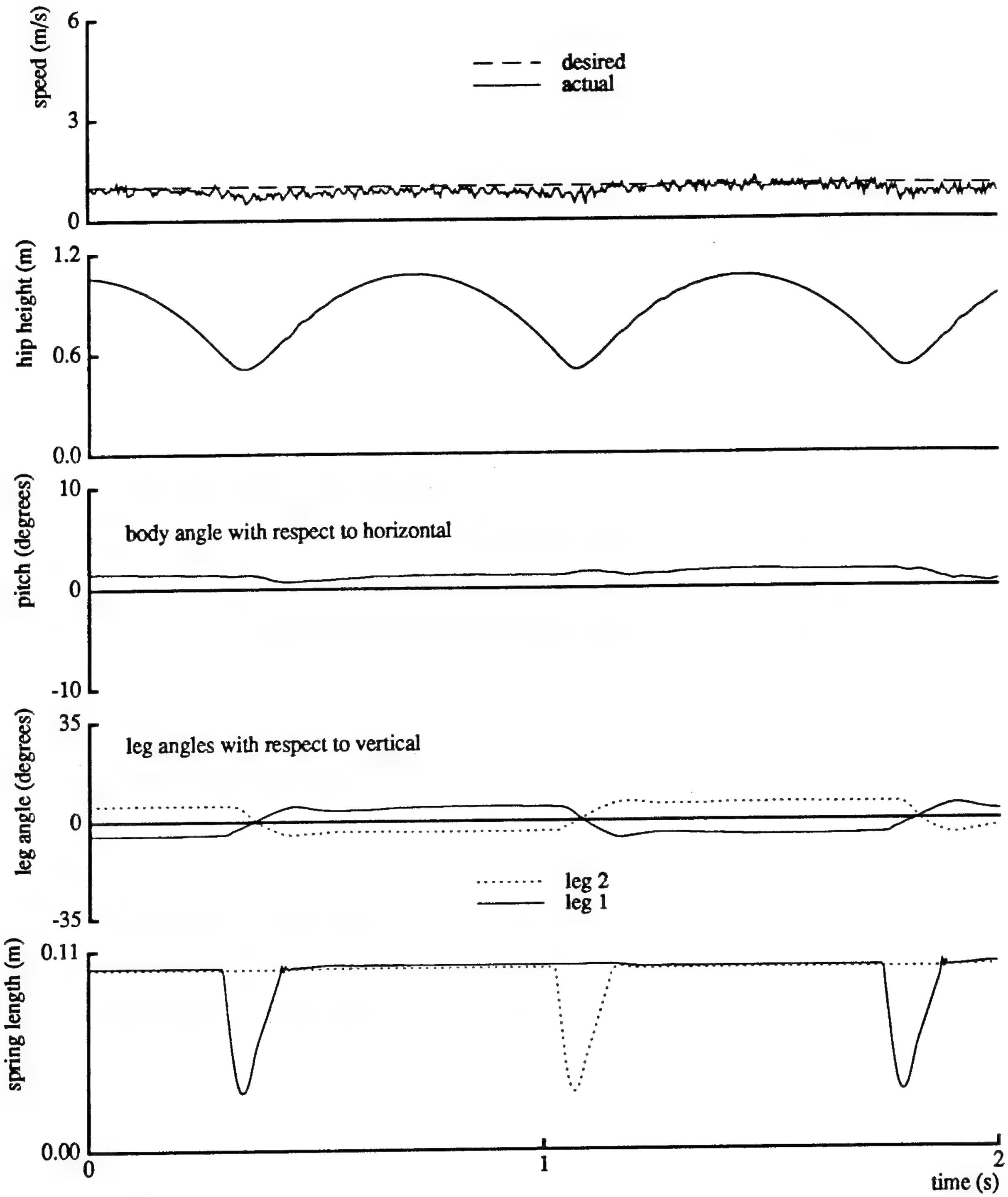


Figure 4–7: The planar biped ran forward at 1 m/s. The top three graphs show the forward speed, hip height, and pitch angle of the body. The fourth graph shows the equal and opposite motions of the two legs as they sweep back and forth. The bottom graph shows how the two leg springs compress as the biped bounces alternately off of the two feet. (Data file B88.241.4)

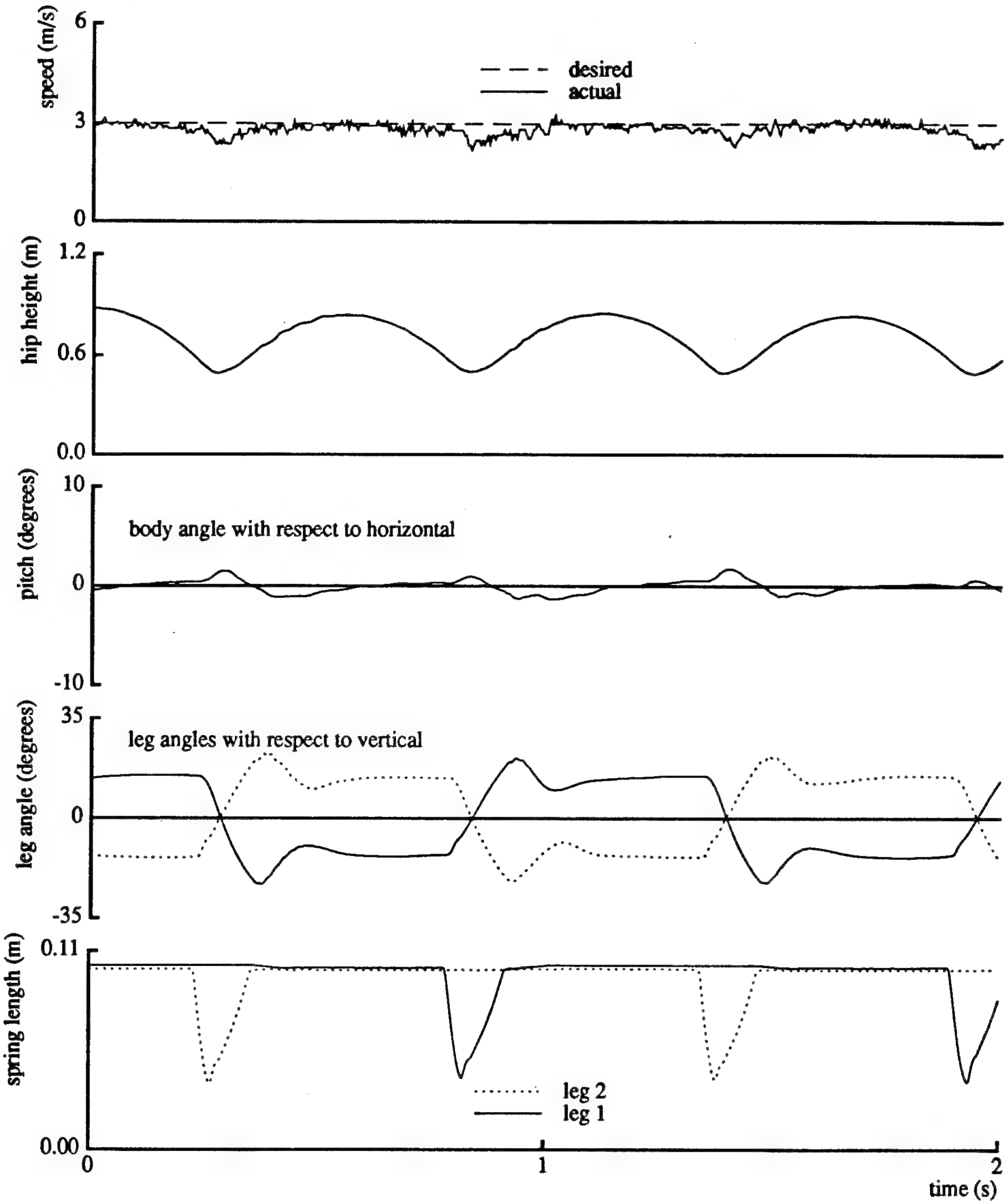


Figure 4–8: The planar biped ran forward at 3 m/s. As in figure 4–7, the top three graphs show the motion of the body, and the bottom two graphs show the angle and compression of the legs. The biped took steps more frequently, swept its legs through a larger angle, and took shallower bounces while running 3 m/s than it did when running 1 m/s. (Data file B88.241.3)

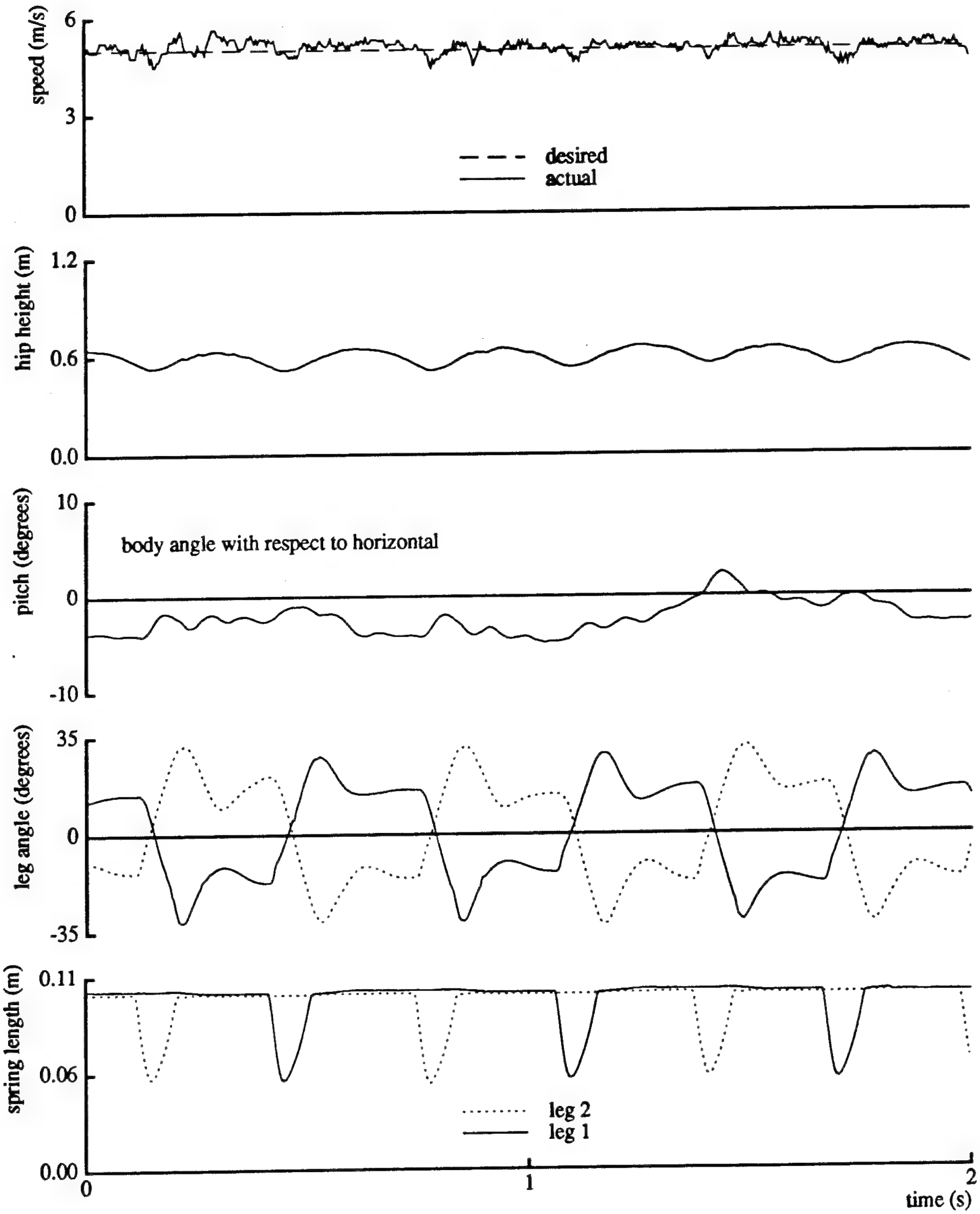


Figure 4–9: The planar biped ran at 5 m/s. In this run, the legs swung through a much larger angle and the hip oscillations were much smaller than in the slower runs. (Data file B88.241.1)

Step Parameters at 1, 3, 5 m/s

commanded speed (m/s)	1.0	3.0	5.0
observed speed (m/s)	0.90	2.92	4.99
stance duration (s)	0.128	0.128	0.088
flight duration (s)	0.588	0.392	0.320
step period (s)	0.716	0.520	0.320
stance travel (m)	0.10	0.34	0.43
flight travel (m)	0.55	1.18	1.17
step length (m)	0.65	1.52	1.60
leg length (m)			
touchdown	0.595	0.595	0.591
liftoff	0.666	0.658	0.620
leg angle w.r.t vertical (°)			
touchdown	-2.2	-11.1	-19.5
liftoff	7.4	21.9	25.2
body angle w.r.t horizontal (°)			
touchdown	1.3	0.0	-4.1
liftoff	1.3	-0.2	-2.5
vertical velocity (m/s)			
touchdown	-2.95	-2.38	-1.60
liftoff	3.00	2.12	1.54
horizontal velocity (m/s)			
touchdown	0.75	2.58	5.02
liftoff	0.86	2.97	4.97
data file	B88.241.4	B88.241.3	B88.241.1

Table 4–3. Each column of data is for a typical step on leg two of the planar biped while it was running at constant speed. Increasing speed was accompanied by longer steps and shorter step periods, as shown by these data and the graphs in figure 4–10. During these experiments, the air pressure in the leg springs was 90 psi and the thrust algorithm extended the leg actuator as quickly as possible during stance.

When the biped ran slowly, the hip joints did not move very much during flight. After liftoff, the control system servoed the legs into position for the next touchdown, and then the system waited for the foot to hit the ground. When the biped ran faster, these waiting periods were shorter, because it took longer to get the legs into position, and the flight duration was shorter. The angular momentum of legs at liftoff was higher for fast running, so it took more time for the hip actuators to decelerate the legs and move them into position for landing.

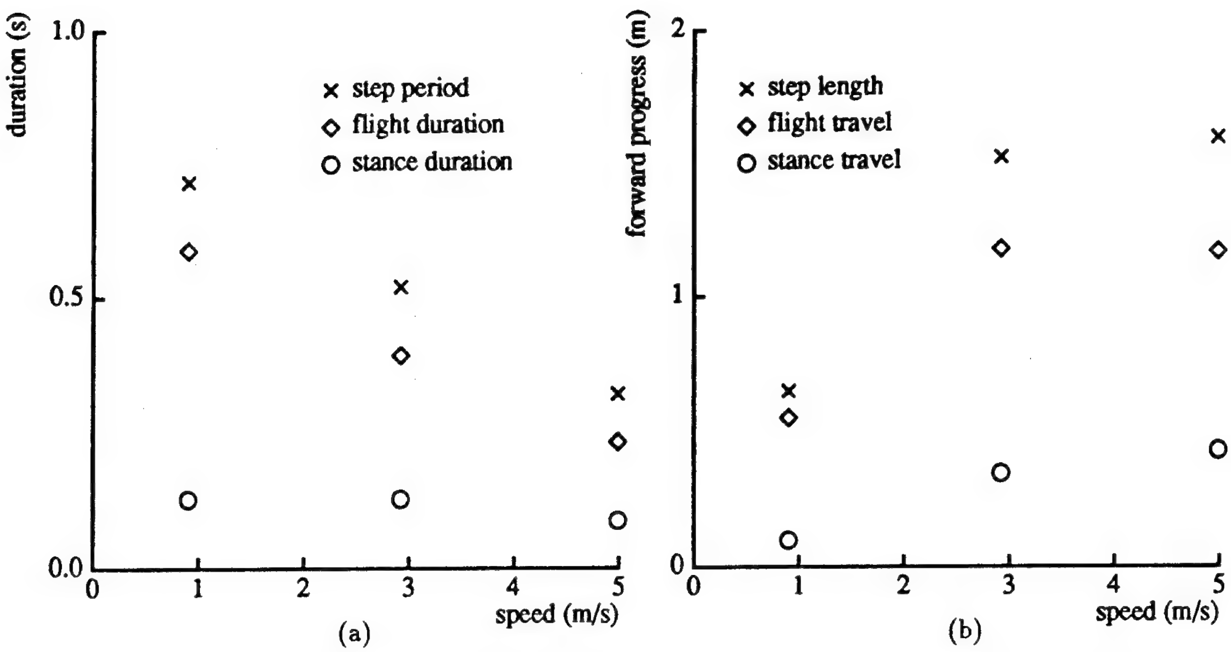


Figure 4–10: These graphs show data from table 4–3. (a) Each step takes less time when the biped running fast than when it is running slow slowly, with most of the difference caused by reduced flight duration. (b) The biped takes longer steps step during fast running than during slow running. The increase in step length is much greater from 1 m/s to 3 m/s than it is from 3 m/s to 5 m/s.

4.6 Leg Length

The longer a legged system's legs are, the faster it can run. Table 4–4 and figure 4–11 show the results of nine experiments with the planar biped, each with a different leg length. During flight, the control system servoed the active leg to the the indicated length. During stance, the leg actuator extended, so the leg was longer at liftoff than it was at touchdown. During each experimental run, I used a joystick to increase the desired running speed, attempting to find the highest speed at which the biped would run without losing its balance. The reported speed for each run is the highest average speed for one lap of the 16 m running track.

The control system could select leg lengths between 0.50 m and 0.65 m by adjusting the leg actuator. For longer leg lengths, the biped's legs were extended with stilts and joined to the bottom of the legs. The feet were moved to the bottom of the stilts. A 0.191 m stilt gave a leg length of 0.844 m, and a 0.391 m stilt gave a leg length of 1.005 m.

Figure 4–11 shows that the top running speed of the biped increased with increasing leg length, but may flatten out above 0.8 m.

The increasing leg length of the biped was not accompanied by other changes specified by any principle of geometric or elastic similarity. No dimension other than leg length

The Effect of Leg Length on Running Speed

leg length touchdown (m)	leg length liftoff (m)	running speed (m/s)	data file
0.503	0.518	3.30	B88.23.15
0.526	0.541	3.89	B88.23.14
0.550	0.561	4.34	B88.23.13
0.573	0.586	4.41	B88.23.12
0.597	0.611	4.61	B88.23.11
0.619	0.636	3.93	B88.23.9
0.643	0.659	4.36	B88.23.8
0.797	0.844	5.88	B89.24.6
0.982	1.005	5.83	B88.152.1

Table 4–4. The planar biped ran nine times, each time with a different leg length. During flight, the control system adjusted the length of the leg. The leg actuator extended during stance, so the leg was longer at liftoff than at touchdown. The desired extension was 0.021 m, but the leg typically left the ground before the actuator finished extending. In each run, the experimenter raised the forward running speed to the highest value that could be maintained without the biped losing its balance. The listed speed is the average for a complete circuit around the 16 m circular track. Initially the range of possible leg lengths was 0.50 m to 0.65 m. Longer leg lengths were obtained by adding stilts to the end of the biped's legs. A 0.191 m stilt gave a leg length of 0.844 m, and a 0.391 m stilt gave a leg length of 1.005 m. During the experiments with the stilts, the leg actuator extended as fast as possible during stance, rather than trying to extend 0.021 m as in the other experiments.

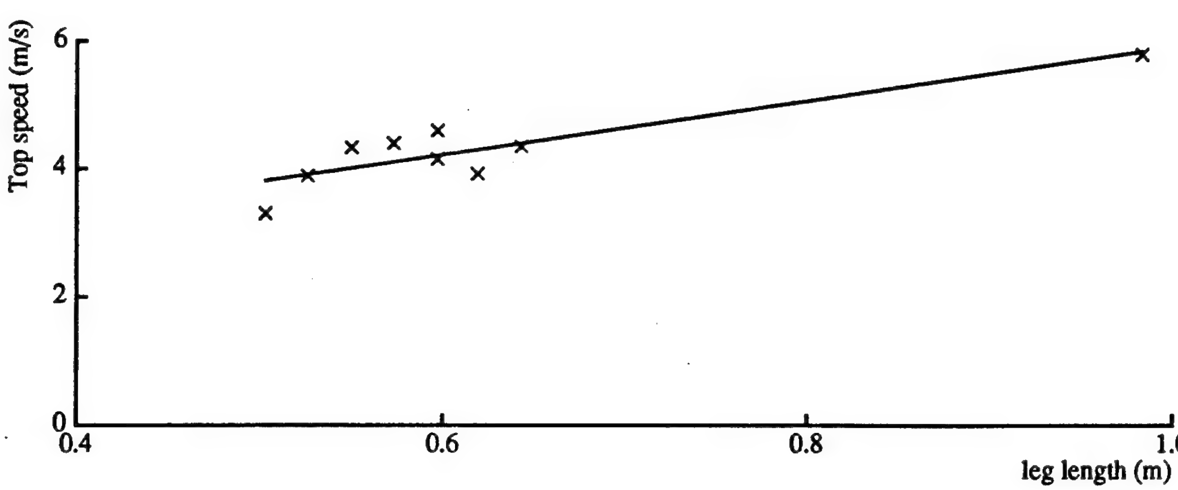


Figure 4–11: The longer the legs were, the faster the planar biped ran. With a leg length of0.844 m, the biped ran 5.9 m/s (13.1 mph), its highest speed ever.

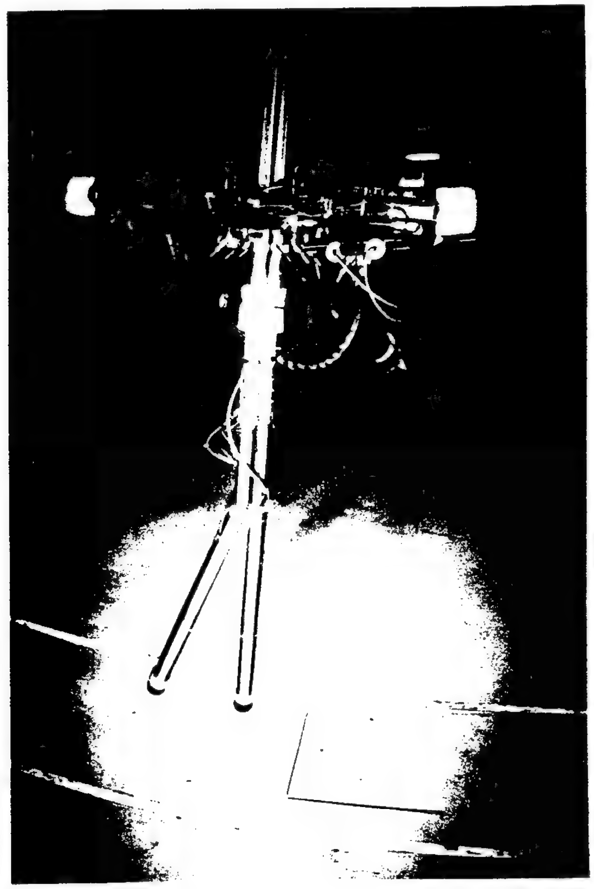


Figure 4–12: One of the long stilts broke after several running experiments. The wall of the tubing tore where it was screwed onto the plug that joined the stilt to the leg tube. Long legs are more vulnerable to buckling failure than short legs.

changed. Since the diameter of the legs did not change, the strength of the legs remained the same, and the factor of safety against structural failure got smaller as the legs got longer. Figure 4–12 one of the consequences. After several running experiments at the longest leg length, one of the stilts broke where it was attached to the leg. The bending force on the leg had torn the stilt where it was fastened to the leg.

The leg springs did not get any longer when the legs got longer, and that may have caused a problem. If the body were to move horizontally during stance, then the leg length when the leg was vertical would be $\ell\cos(\theta)$, where ℓ and θ are the leg length and leg angle at touchdown. So when the leg was vertical, the leg spring would be deflected at least $\ell[1-\cos(\theta)]$ from its length at touchdown. The actual deflection of the spring would be greater, since the path of the body is concave upward, so that the hip is always lower in the middle of stance than at touchdown or liftoff. The air springs on the biped have a maximum deflection of less than 0.10 m. For a leg angle of 25°, a leg length of 0.66 m would require the spring to deflect at least 0.06 m, but for a leg length of 0.98 m, the required deflection

would be more than 0.09 m. The limit on leg spring deflection probably prevented the biped from using its full hip travel when it was running with the long stilts.

When the long stilts were on the machine, the hip servo position gain had to be reduced to keep the servo from oscillating. Lengthening the legs increased their moment of inertia and their flexibility, both of which lowered the natural frequency of the first mode of vibration of the leg. Lowering the position gain softened the servo so that it did not excite the vibration of the leg.

The planar biped runs faster with long legs than with short legs. The broken stilt and the need to soften the hip servo point out some of the problems that accompany increasing leg length.

4.7 Leg Stiffness

The stiffer the leg springs are, the faster a legged system can run. Table 4–5 and figure 4–13 show the results of eight experimental runs with the planar biped, each at a different leg stiffness. Before each experimental run, I set the indicated pressure with the regulator that supplies air to the pneumatic leg springs. During the runs, I used a joystick to gradually increase the desired running speed to find the highest speed at which the biped would run without losing its balance. The reported speed for each run is the highest average speed for one lap of the 16 m circumference running track. The reported stance duration is the average of the stance durations observed during the fastest lap.

The leg stiffness of the planar biped depends on how much air is in the leg springs. An air line leads from the spring chamber to a regulator that maintains the desired pressure in the line. A check valve isolates the spring chamber when pressure inside is higher then the pressure in the line. If air leaks out of the spring while it is compressed, then when the spring extends the pressure inside drops below the pressure in the air line, and air flows past the check valve to bring the spring back up to the desired pressure. The higher the air pressure is, the more air there is inside of the leg spring. That increases both the stiffness and the preload force of the spring.

Figure 4–13 shows that the biped ran faster and took steps with shorter stance duration when it was running with high leg spring air pressure than when it was running with low leg spring air pressure. The stiffness of the legs, and thus the natural frequency of the bouncing motion increased with the pressure.

4.8 Body Attitude

Body attitude is important to top running speed because of the limited range of motion of the hip joints. If the body tips forward during flight, the hip joint limit prevents the foot from reaching as far forward for landing as it can with a level body. The distance that the

The Effect of Air Pressure on Stance Duration and Running Speed

resting pressure (psi)	stance duration (s)	forward speed (m/s)	data file
10	0.150	3.16	B88.026.6
20	0.130	4.04	B88.026.7
30	0.120	4.62	B88.026.8
40	0.116	4.45	B88.026.9
50	0.106	4.84	B88.026.10
60	0.100	4.99	B88.026.11
70	0.093	5.40	B88.027.0
80	0.088	5.42	B88.027.2

Table 4–5. The planar biped ran eight times, each with a different air pressure in the leg springs. In each run, the experimenter raised the forward running speed to the highest value that could be maintained without the biped losing its balance. The reported forward speed, stance duration, and vertical landing velocity are average values for a complete circuit around the 16 m circular running track. During these runs, the leg length at touchdown was 0.623 m, and the thrust algorithm extended the leg actuator as rapidly as possible during stance.

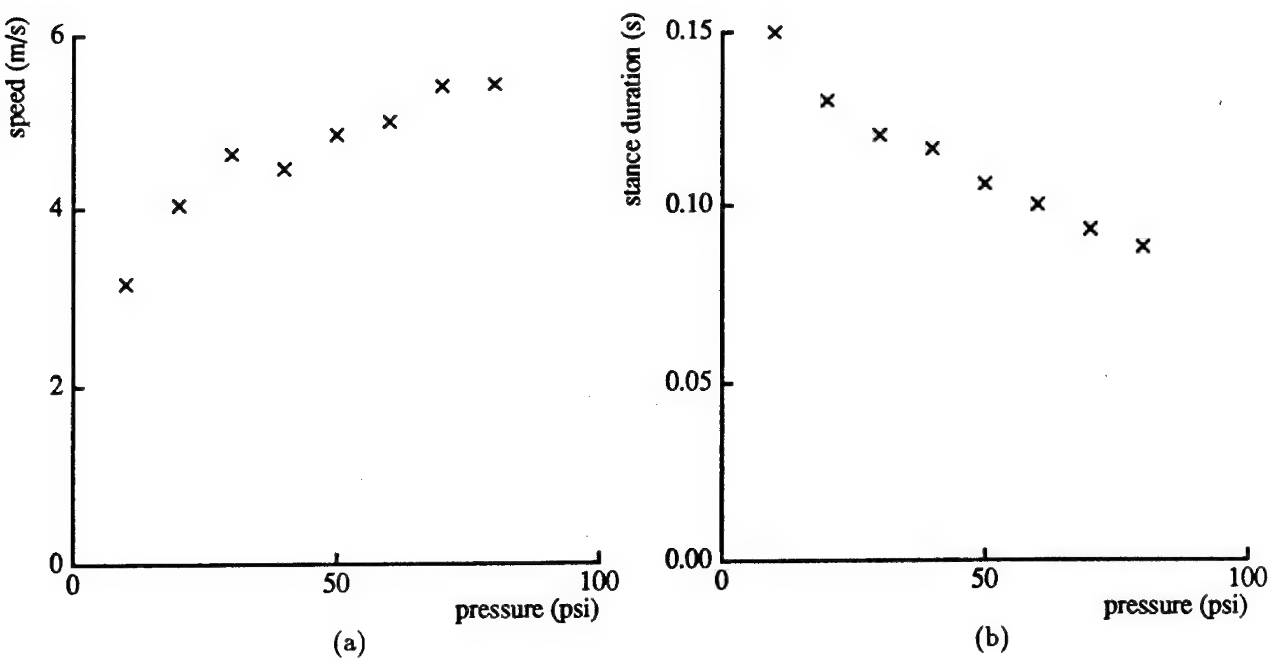


Figure 4–13: These plots of data from table 4–5 show the variation of top running speed (a) and of stance duration (b) as a function of the air pressure in the leg springs. Each point represents the average stance duration or forward speed over the fastest lap at the given air pressure.

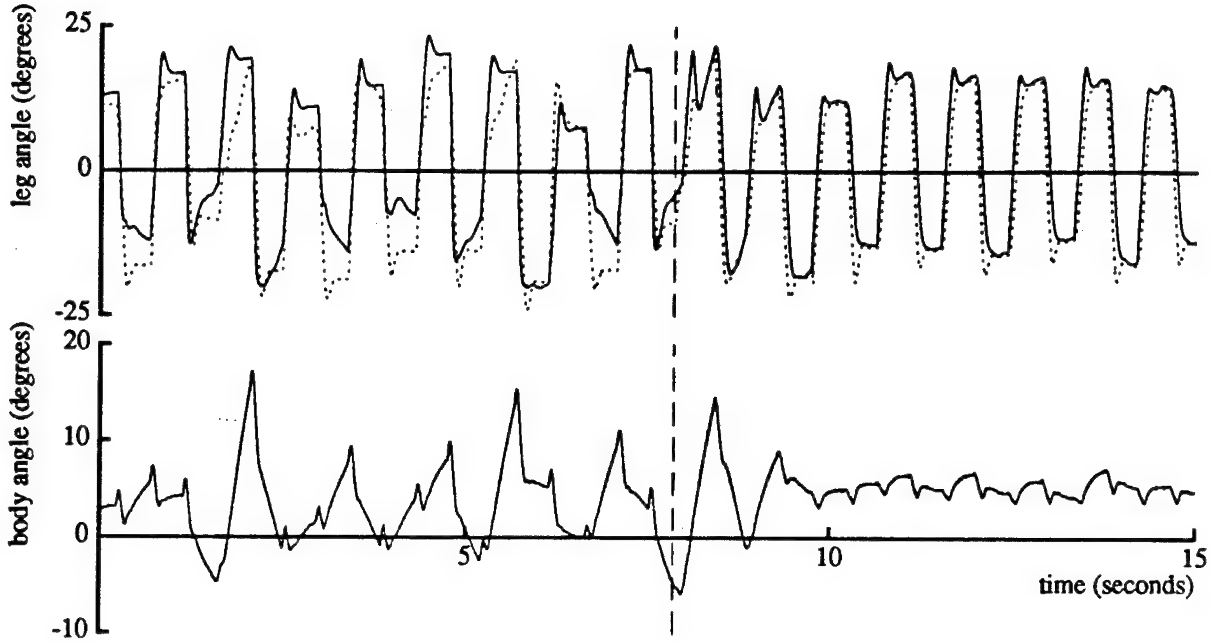


Figure 4–14: A control algorithm that kept the leg angles equal and opposite reduced the the amplitude of the oscillations in body attitude. The top graph shows the angle of each leg with respect to the axis of symmetry of the body. The sign of the angle of leg 2 is reversed so that when the leg angles are symmetric the lines are on top of one another. The vertical line marks a switch from an algorithm that moved the legs independently to one that ensured that the leg angles were mirror images. The axis of symmetry was a line passing through the hip joint perpendicular to the body. The bottom graph shows that changing the leg positioning algorithm reduced the oscillations in body angle from about 20° peak-to-peak to about 6° peak-to-peak. In this experiment the planar biped was running about 2.5 m/s (5.6 mph). (Data file B87.325.3)

foot needs to reach forward for landing is proportional to running speed, so forward tipping of the body reduces the top running speed. If the body tips backward during stance, the hip joint limit prevents the foot from reaching as far backward as it can with a level body. In this case, the hip may reach the joint limit before the leg leaves the ground, causing a sudden forward pitching of the body. Top running speed requires good control of body attitude so the hip joint can sweep through its full range of motion during stance. If there were no kinematic limits to hip angle, then body attitude would not affect running speed.

Mirroring

Hip torques that position the legs also rotate the body. The control system can minimize disturbances to the body attitude by ensuring that the two legs move at the same time and in opposite directions. During stance, while the support leg sweeps backward, the other leg swings forward. During flight, while one is positioned for landing, the other leg makes compensating motions to reduce the torques on the body. This mirroring action

substantially reduces the variation in body attitude that occurs when each leg is moved independently.

Figure 4–14 shows the result of an experiment comparing two different algorithms for moving the idle leg. At the time marked by the vertical dotted line, the control system switched algorithms. Before that time, the legs were positioned independently, and afterwards they were positioned according to the mirroring algorithm. The mirroring algorithm reduced the oscillations in pitch angle, from about 20° peak-to-peak to about 6° peak-to-peak.

When the two legs were being positioned independently, the algorithm was as follows: during stance, the stance leg was swept back by the forward motion of the body, and by hip torques selected by the body attitude control servo. A leg angle servo moved the swing leg forward into position for the next touchdown. During flight, leg angle servos positioned the leg that would touch down next for landing, and servoed the leg that had just lifted off to the angle it had at liftoff. The leg angle servos were as stiff as possible, in order to minimize steady state error. The swing leg moved forward very quickly and stopped at the desired position well before the end of stance. After liftoff, the leg that had just left the ground was still rotating, so the servo applied torques to stop the rotation and move the leg back to the position it had at liftoff. These torques disturbed the body attitude.

The mirroring algorithm servoed the idle leg so that its hip angle was equal and opposite to that of the active leg. During stance, the support leg was active, and swept back as the body moved forward. The swing leg was idle, and moved forward at the same rate as the support leg moved back. During flight, the leg that would next touch down was active, and was positioned for landing as usual. The leg that had just lifted off was idle, and made motions symmetric to the active leg. Compared with the independent positioning algorithm, mirroring caused the swing leg to advance more slowly, which reduced the disturbance to body attitude. During flight, mirroring caused the reaction torques generated by positioning one leg to be canceled by torques from moving the other leg.

Sweep Compensation

The hip actuators on the planar biped have internal damping that causes a velocity dependent discrepancy between the commanded force and the delivered force. When the biped runs fast, it leans forward until the body attitude servo commands enough force to overcome the actuator damping. Explicitly compensating for the velocity dependent forces reduces the tendency to lean forward.

Figure 4–15 shows the body angle during an experimental run in which the control system gradually increased the running speed from zero to 4 m/s. The graphs show that the average body angle increased as running speed increased.

When the biped runs in place, the hips barely move. When it runs forward, each hip rotates one way as the leg sweeps back during stance and the other way as the leg swings forward in preparation for the next step. These hip joint velocities are proportional to running speed. Each biped hip actuator behaves like a torque source in parallel with a

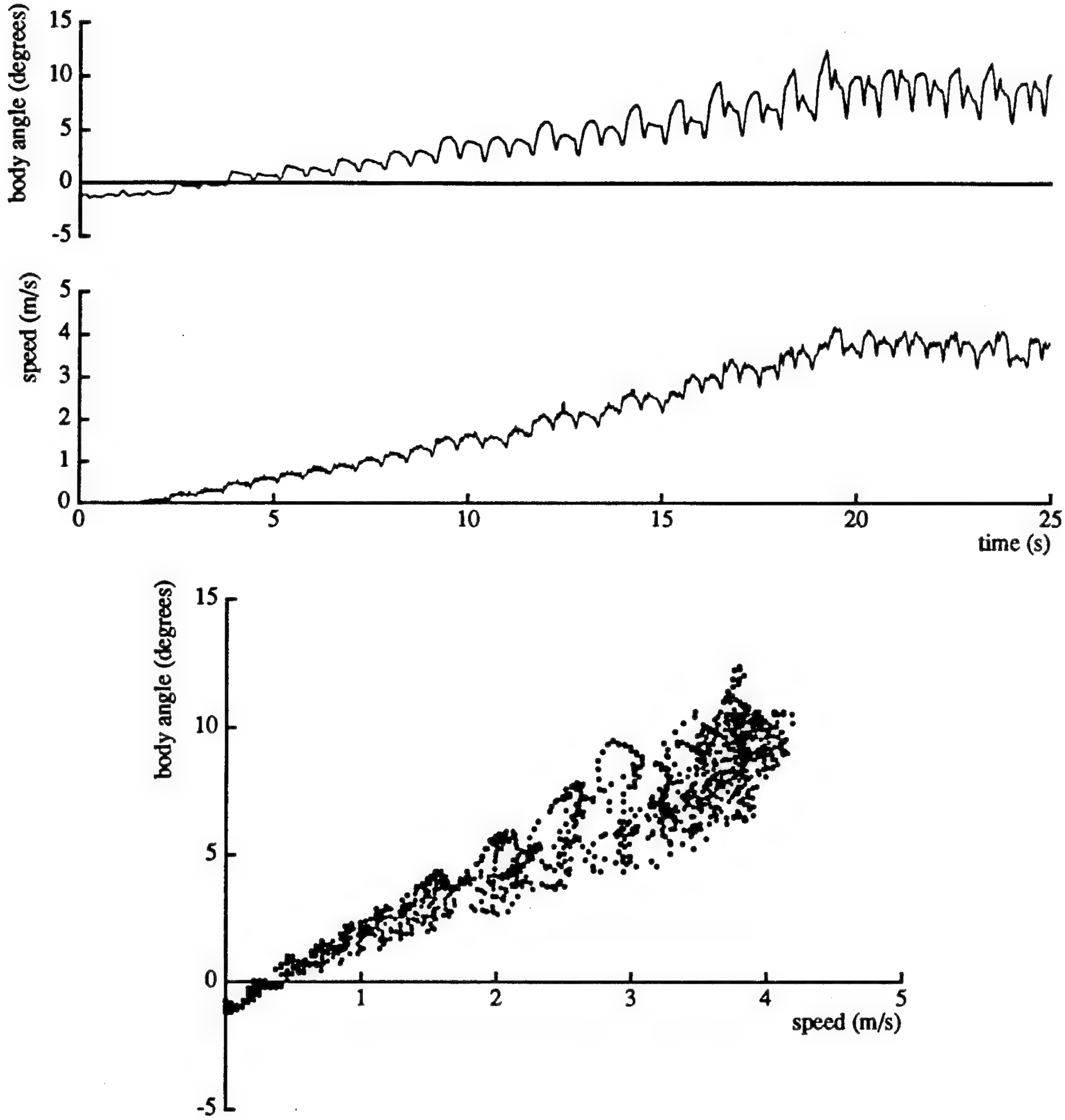
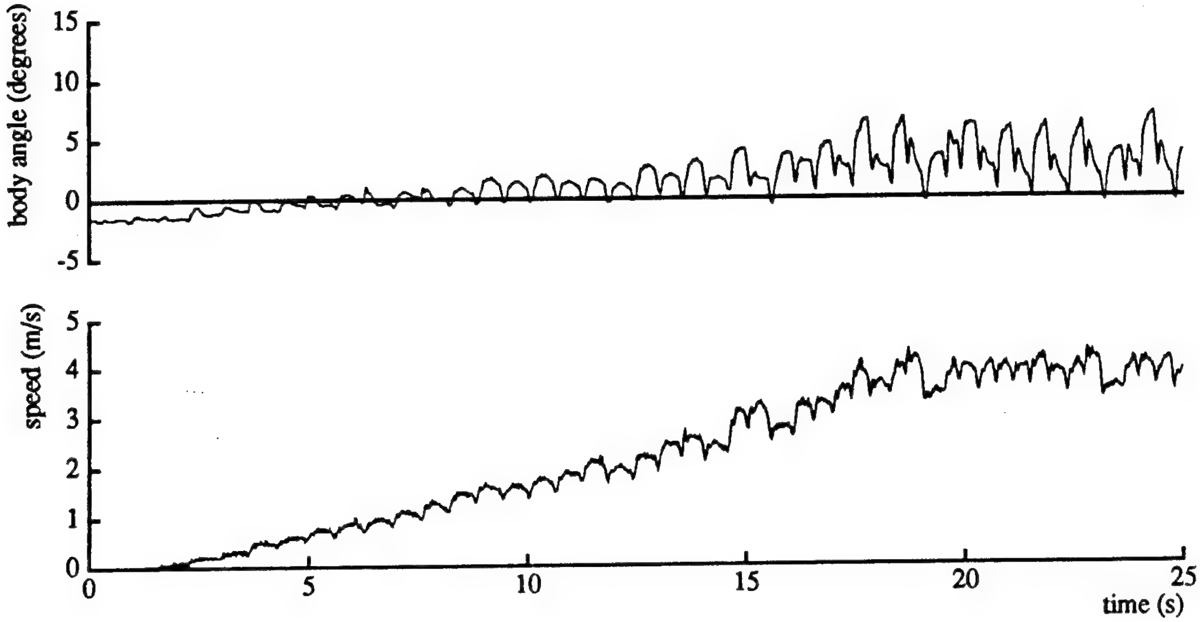


Figure 4–15: The planar biped leans forward when it runs fast. The upper graph shows the angle of the body with respect to horizontal. The middle graph shows the forward running speed. The bottom graph shows body angle plotted as a function of running speed. The body angle oscillated during each step. As the running speed increased, the peak-to-peak amplitude increased with a slope of about 2.0°s/m. Similarly, the average body angle increased with a slope of about 2.2°s/m. At 4 m/s the offset was 7° and the peak-to-peak amplitude was 5°. (Data file B89.6.2)



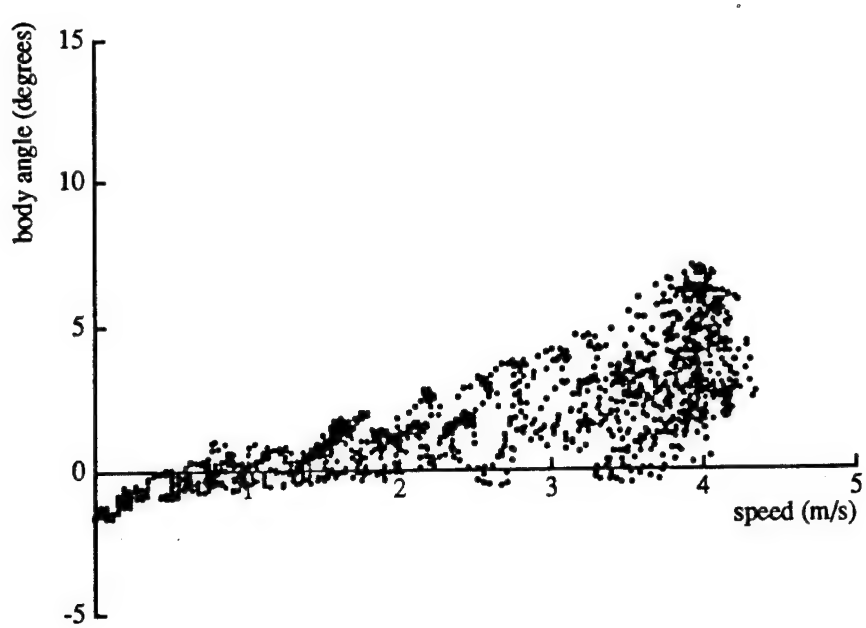


Figure 4–16: Compensating for velocity dependent forces in the hip actuators reduced the average body angle, but did not reduce the amplitude of oscillation. As in figure 4–15, the body angle took on an offset and an oscillation as the running speed increased. Sweep compensation reduced the average body angle but not the amplitude of the oscillations. The increase in offset was about 1.5°s/m. At 4 m/s the offset was about 3° and the peak-to-peak amplitude was about 6°. (Data file B89.16.3)

damper. For a given input signal, the actuators produce less force when they are moving quickly than when they are moving slowly. The damping forces are proportional to hip rotation rate, which is proportional to running speed.

The hip torque to overcome the actuator damping comes from the body attitude servo. During stance, the servo applies torques proportional to the angle and angular velocity of the body. The angle stabilizes when the body has leaned forward enough that the attitude servo generates a correcting torque equal to the torque caused by the actuator damping force. Because the force is proportional to running speed, body leans farther forward as the biped runs faster.

Compensating for the velocity dependent torques reduces the lean of the body. Figure 4–16 shows the body angle during an experimental run in which the body attitude servo added negative damping to the hip actuators by feeding back a signal proportional to the actuator velocity. The signal to the servovalve was:

$$\tau = k_\rho \rho + k_{\dot\rho} \dot\rho + k_{\dot w} \dot w, \tag{4.17}$$

where ρ and $\dot\rho$ are the pitch angle and the pitch rate, $\dot w$ is the hip actuator velocity, and k_ρ, and $k_{\dot\rho}$ are the position and velocity gains that control pitch, and $k_{\dot w}$ is the inverse damping coefficient. The modification reduced the body angle offset from about 7° to about 3°. The same result might have been obtained by adding an integral term to the body attitude servo.

4.9 Power Dissipation

The planar biped dissipates slightly more power when it runs fast than when it runs slowly. Figure 4–17 shows the power dissipated during an experimental run in which the control system gradually increased the running speed from zero to 4 m/s. The peak power was nearly constant, increasing very slightly at 4 m/s. The average power increased gradually as running speed increased. The instantaneous power shown is the product of the supply pressure measured by a sensor on the robot, and the total flow computed from the actuator velocities. The flow computation included an estimate of the flow that leaked through the servovalves, which does not appear in the actuator velocities. The estimated leakage was about 39 cc/s, which at a system pressure of 3000 psi corresponds to 0.8 kW. The hydraulic pump maintained a nearly constant pressure, so the power was proportional to the flow.

A 7.5 kW motor drives the hydraulic pump. The peak instantaneous power of 6.6 kW probably exceeds what the pump can deliver. However, the 5 gal hydraulic accumulator, the compliance of the hydraulic hoses, and the inertia of the oil all filter out flow transients, so the pump never has to deliver the peak power. The average power dissipation was less than 3 kW, which is well within the capacity of the pump and motor. The biped's running speed is not currently limited by the available power.

Figures 4–18 and 4–19 show the hip angles and power dissipation during a pair of experiments in which the planar biped ran using two different gaits. In the first experiment,

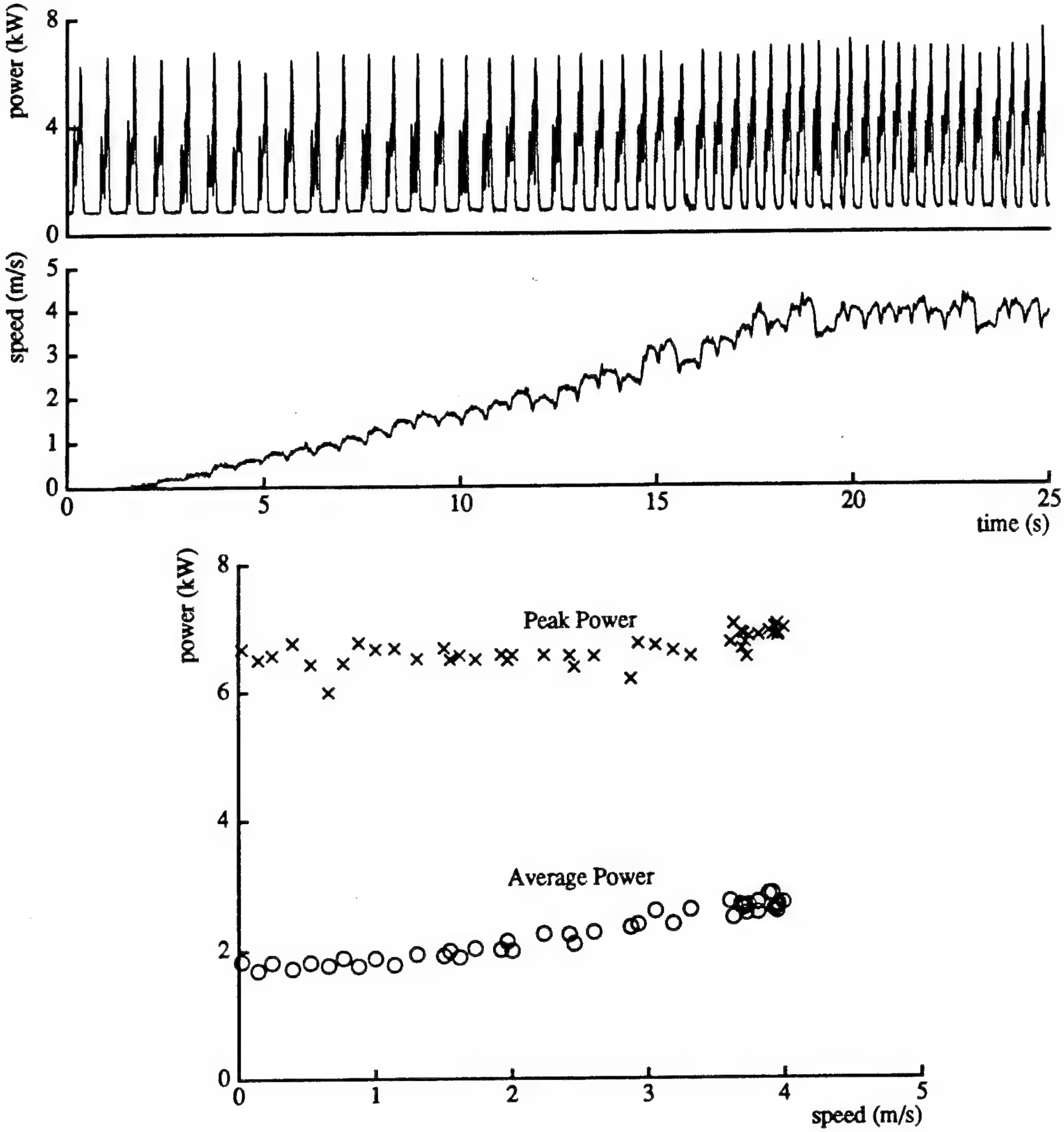


Figure 4–17: The planar biped dissipates slightly more energy to run fast than it does to run slowly. The top two graphs show the instantaneous power and the running speed as the biped accelerated from rest to 4 m/s. The bottom graph shows power plotted as a function of running speed. The crosses show the peak instantaneous power on each step, and the circles show the average power for each step. The peak power was a nearly constant 6.6 kW, increasing very slightly at speeds of about 4 m/s. The average power increased gradually, from 1.8 kW for hopping in place, to 2.7 kW for running 4 m/s. (Data file B89.16.3)

the biped ran with its usual alternating two-legged gait. In the second experiment, it ran by hopping on one leg. The second leg stayed short, and moved back and forth to compensate for the reaction torques of the active leg.

The biped ran at about the same speed with both gaits, but dissipated less energy when it ran on only one leg. It ran 5.3 m/s on two legs, and dissipated 3.5 kW, with instantaneous peaks of 7.2 kW. On one leg, it ran 5.4 m/s, dissipated 2.9 kW, with instantaneous peaks of 6.0 kW.

Running on one leg required the legs to sweep back and forth twice as frequently as they did in two-legged running, so the hip actuators dissipated more power. On the other hand, running on one leg meant that the other leg never had to change length. The leg actuator has a large area and a long stroke, so moving it causes a large flow that dissipates a lot of power without doing any work. Keeping one leg short and not moving its actuator saved more than enough energy to compensate for the increased dissipation of the hip actuators.

The biped's hydraulic system is very inefficient for applying small forces at high velocities. The pump supplies oil at constant pressure, so the power supplied is proportional to the flow of oil. To apply a small force, an actuator throttles the oil down to a lower pressure, dissipating energy. That the biped dissipated less power running on one leg than running on two is an artifact of the constant pressure hydraulic system.

4.10 Summary

The running speed of a legged system depends upon the frequency and length of its steps. The time required for a step can be reduced by stiffening the legs, and the step length can be increased by lengthening the legs. If body attitude is not well controlled, the limited range of motion of the hips limits the length of the steps.

Experiments with the planar biped showed that it runs faster with stiff legs than with soft legs, and that it runs faster with long legs thatn with short legs. To get it to run fast, the control system reduces variation of body attitude by moving the legs symmetrically, and by compensating for velocity dependent hip actuator forces. The biped's power dissipation is well within the capacity of its power supply.

Figure 4–20 shows the leg and body motions of the planar biped during its fastest run. It ran 5.9 m/s (13.1 mph) on long, stiff legs. The leg length at landing was 0.844 m, and the air spring pressure was 85 psi. The control system moved the legs symmetrically, and compensated for hip actuator damping forces.

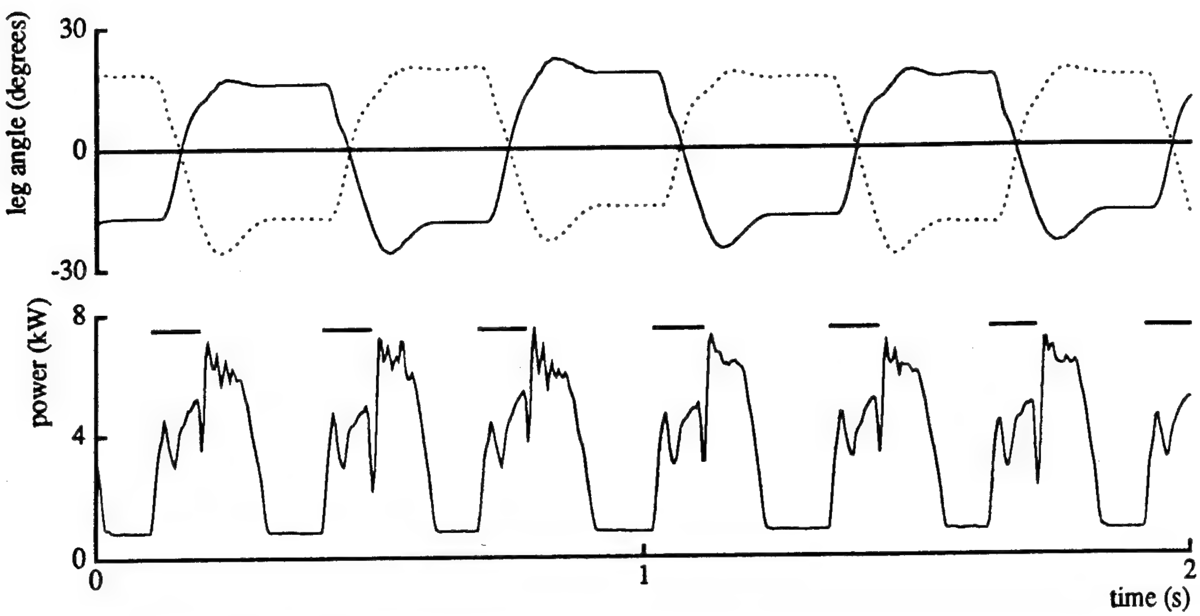


Figure 4–18: The graphs show the leg angles and instantaneous power dissipation while the biped ran 5.3 m/s. The biped dissipated 3.5 kW, with instantaneous peaks of 7.2 kW. The power shown is the product of the total flow with the nominal system pressure of 3000 psi. The horizontal bars indicate the stance phases. The peak power dissipation occurred just after liftoff, when both legs were changing length. (Data file B86.288.20)

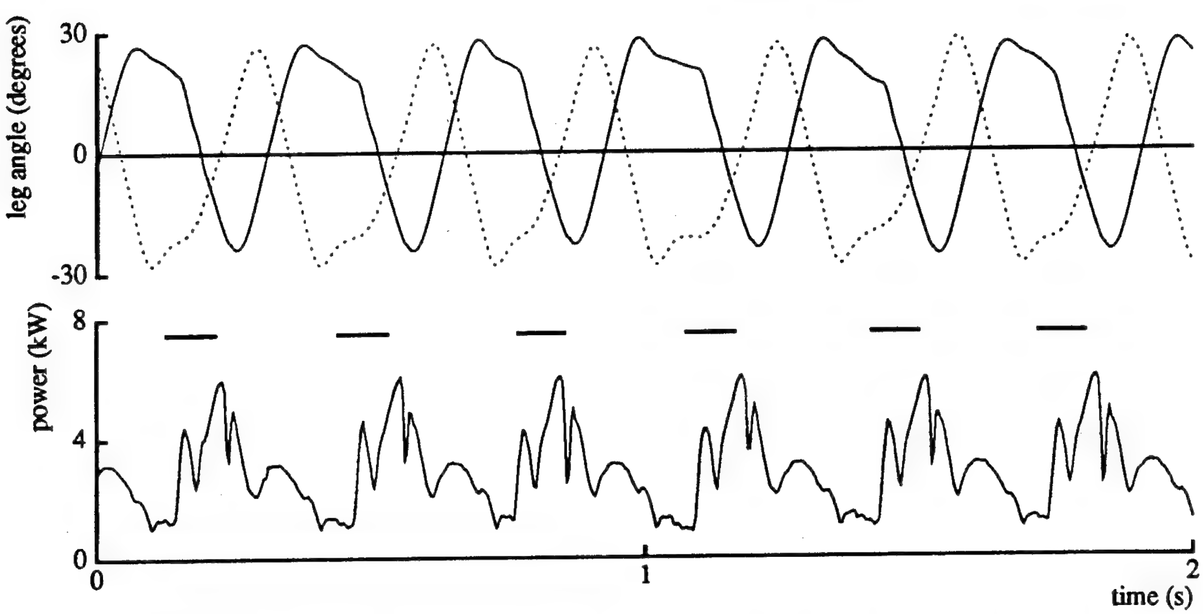


Figure 4–19: While running 5.4 m/s on one leg, the biped dissipated 2.9 kW, with instantaneous peaks of 6.0 kW. Running on one leg dissipated less power than running on two legs because one of the legs remained short, so that it did not dissipate any power by changing length. (Data file B86.288.20)

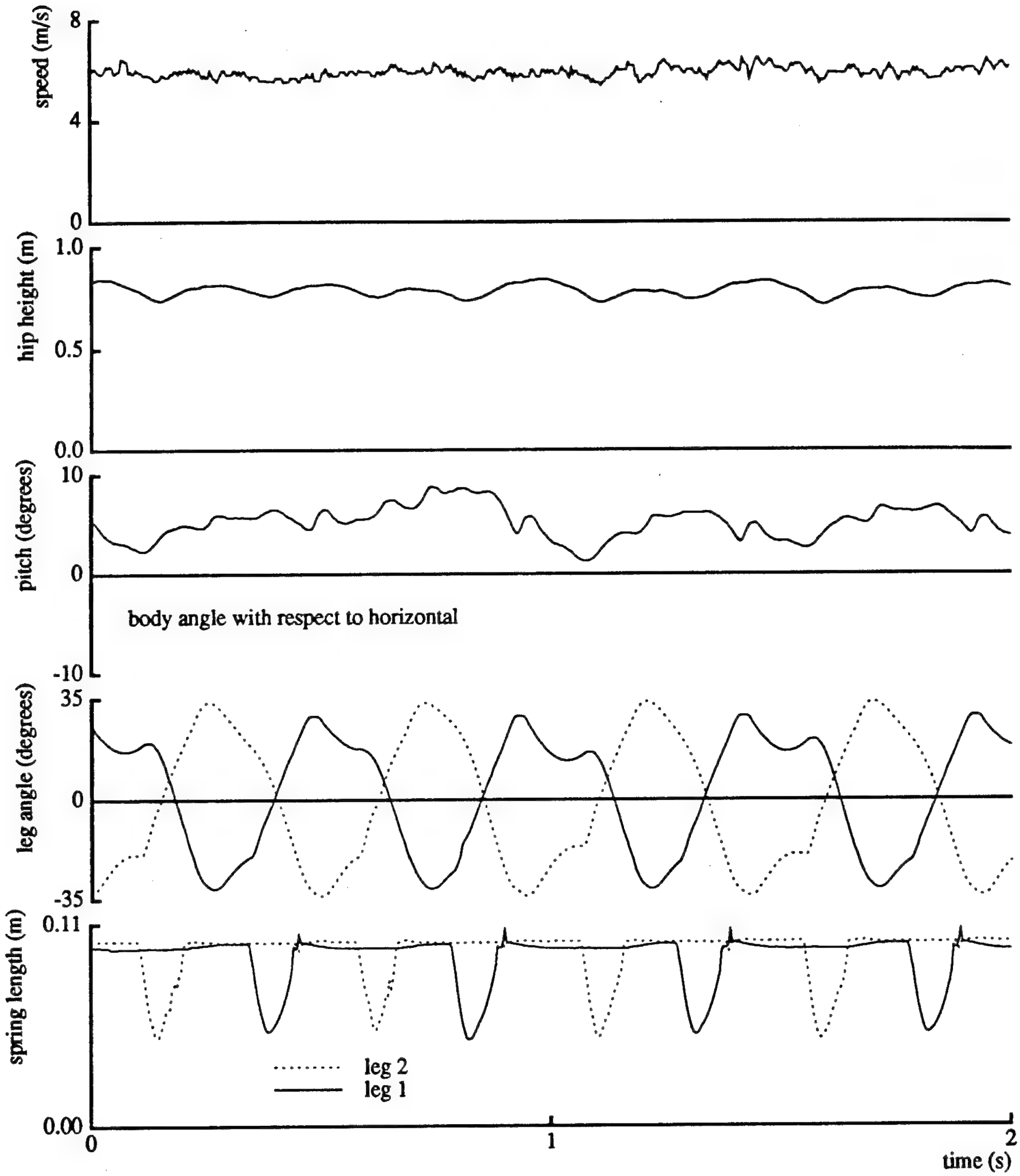


Figure 4–20: The biped ran 5.9 m/s (13.1 mph), the fastest run so far. The top three graphs show the forward speed, hip height, and angular orientation of the body. The fourth graph shows the equal and opposite motions of the two legs as they sweep back and forth. The fifth graph shows how the two leg springs compress as the biped bounces alternately off of the two feet. For this run 0.191 m long stilts extended the biped's legs. The legs were 0.797 m at touchdown. The leg actuator lengthened as rapidly as possible during stance, so the legs were 0.844 m long at liftoff. (Data file B89.24.6)

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Chapter 5

Biped Gymnastics

5.1 Abstract

In this paper we examine the nature of a gymnastic maneuver, the forward flip, and describe how a planar biped running machine executes it. To perform a flip the machine runs forward, thrusts with both legs to jump while pitching the body forward, shortens the legs to tuck once airborne, untucks in time to land on the feet, and then continues running. The control system produces these actions and the resulting flip by issuing a prespecified pattern of actuator commands to the actuators in conjunction with a set of control algorithms used for normal running. When all components of the system are working properly, the machine executes nine out of ten flips successfully.

5.2 Introduction

The forward somersault or flip is a gymnastic maneuver in which the performer runs forward, springs off the ground with both feet, rotates the body forward through 360 degrees, and lands in a balanced posture on one or both feet. See figure 5–1. Human gymnasts can do a forward flip as an isolated maneuver, or as part of a floor routine in which the flip is preceded and followed by other maneuvers. The average teenager can learn to do a forward flip in a few weeks with proper coaching and practice. In the 1988 Summer Olympics, Vladimir Gogoladze of the USSR, did triple back flips in the floor exercise.

Rather than study humans doing flips, we programmed a planar biped running machine to do a flip in the laboratory. To perform the flip the biped machine runs forward, thrusts with both legs to jump while pitching the body forward, shortens the legs to tuck once

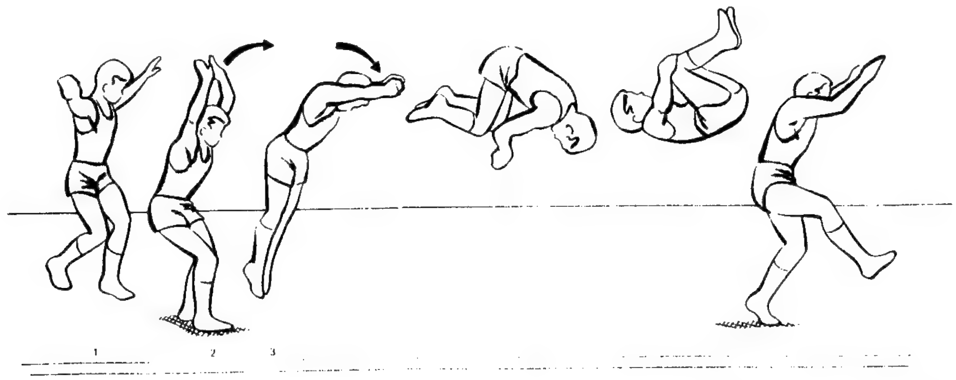

Figure 5–1: Forward flip as performed by a human gymnast. Drawings reprinted from Tonry (1983).

airborne, untucks in time to land on the feet, and then continues running. To develop this program of action, we modified three steps in an otherwise normal sequence of steps. At particular points during the three steps the control system initiated the actions required for the maneuver, such as bringing the legs together, tucking the legs, etc. The timing and pattern of some actions, like accelerating the body about the pitch axis, were provided by prespecified parameters. In most cases, the algorithms normally used to control running speed, balance, and body attitude remained in effect, provided they did not interfere with producing the maneuver. Figure 5–2 is a photograph of the biped taken during a successful flip.

We studied the flip because we are interested in maneuvers that have significant dynamic and ballistic content. The high-jump and baseball pitch are additional examples of such maneuvers. To some degree, this work tests the *motor tape model*, a concept of how humans might produce, store, and modify patterned movement. The motor tape model likens the issuing of neural commands to the playing of a multi-channel tape recorder (Evarts et al. 1970). We wanted to provide concrete detail to the motor tape model, and to see if such a mechanism could be used to produce maneuvers. We found that prespecified patterns of actuator output signals could be used to produce flips with a good degree of reliability, when used in conjunction with more conventional algorithms that provided attitude control and balance. We also found that working on flips was lots of fun.

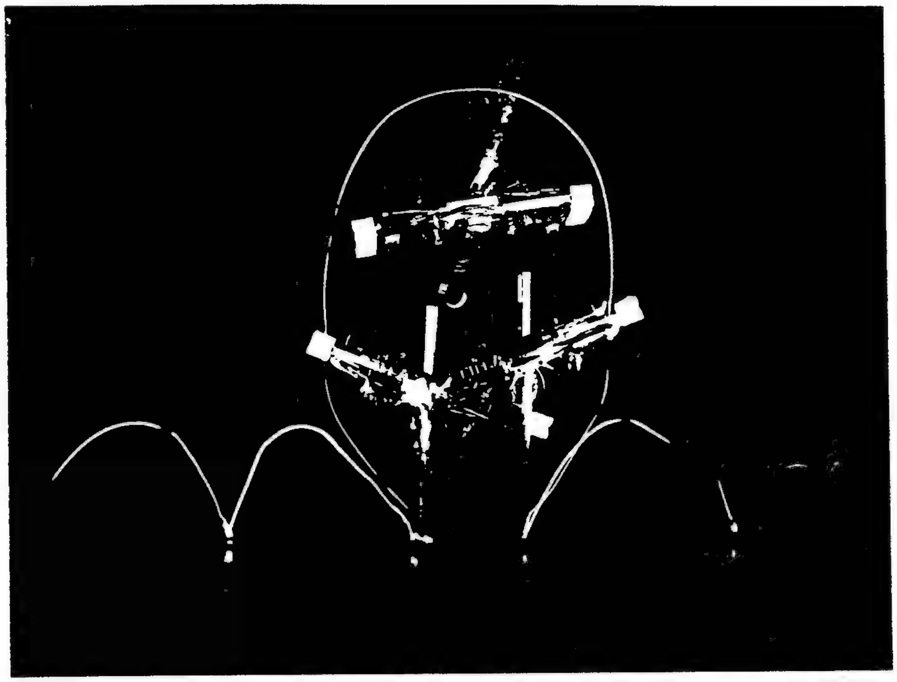

Figure 5–2: Photograph of planar biped doing a flip. Images were taken at liftoff, the top of flight, and at touchdown. The line indicates the path of the close foot. The machine was traveling from right to left.

5.3 Mechanics of the Flip

The planar biped running machine used for the project is shown in figure 5–3. It has two telescoping legs connected to a body by pivot joints that form hips. Each hip has a hydraulic actuator that positions the leg fore and aft. A hydraulic actuator within each leg acts along the leg axis to change the length of the leg, while an air spring makes the leg compliant in the axial direction. The overall motion of the biped is constrained to a plane by a tether mechanism that allows it to move fore and aft, up and down, and to rotate about the pitch axis. The biped running machine is described more fully in Hodgins, Koechling, and Raibert (1985).

A flip is a maneuver in which the body and legs rotate through one or more full rotations during the flight phase. The control must ensure that the system neither over-rotates nor under-rotates. A basic equation governing the behavior of the body during the flight phase of a flip is

$$n\pi = \frac{\dot{\phi}\dot{z}}{g}, \qquad (5.1)$$

where

n is the number of full pitch rotations of the body,
$\dot{\phi}$ is the pitch rate of the body,
$\dot{z}$ is the vertical velocity of the body at the beginning of the flight phase, and

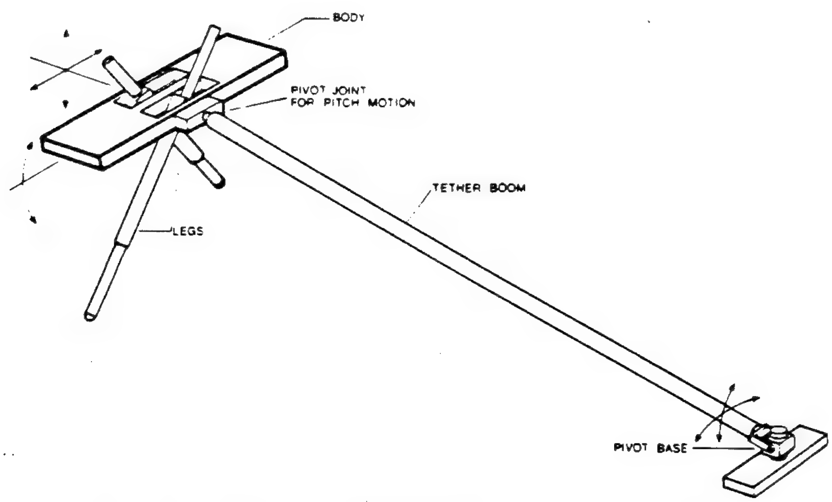

Figure 5–3: Diagram of planar biped used for experiments. The machine travels by running on a 2.5 m radius circle on the laboratory floor. The body is an aluminum frame on which are mounted actuators, hydraulic accumulators, and computer interface electronics. The hip is driven fore and aft by two low-friction hydraulic actuators. Actuators within the legs change the leg lengths and air springs make the legs springy in the axial direction. Onboard accumulators on the hydraulic supply and return lines increase the instantaneous actuator rate. Sensors on the machine measure the lengths of the legs and air springs, the positions and velocities of the hydraulic hip actuators, and contact between each foot and the floor. A tether mechanism constrains the body to move with three degrees of freedom—fore and aft, up and down, and pitch rotation. Sensors on the tether mechanism measure vertical displacement of the body, forward displacement, and pitch rotation. The tether also supports an umbilical cable that carries hydraulic connections, electrical power, and a connection to the control computer. See Hodgins, Koechling, and Raibert (1985) for more details.

g is the acceleration of gravity.

Equation (5.1) relates the vertical velocity of the body to its angular velocity. For n full rotations of the body during the flight phase, the rate of body pitch rotation $\dot{\phi}$, times the duration of the flight phase $2\dot{z}/g$, equals the angular displacement of the body $2n\pi$.

Equation (5.1) relies on several simplifying assumptions. We assume that the legs do not swing with respect to the body during the flight phase, so $\dot{\phi}$ represents the angular rates of both the body and the legs. We further assume that the pitch angle of the body is zero at both liftoff and touchdown, that the altitude of the body is the same at liftoff as it is at touchdown, and that there is negligible rotational friction from the pivot at the end of the boom or wind resistance and, therefore, constant angular momentum during flight.

For (1) we also assume the pitch rate of the system is constant during the flip. Actually, angular rate may change even though angular momentum is constant. For instance, humans reduce their moment of inertia to increase their rotation rate by tucking the arms and legs in close to their bodies. Tucking reduces the moment of inertia by concentrating the masses nearer to the center of mass of the system than when untucked. The ice skater's spin is a dramatic demonstration of this phenomena.

If the angular rate and moment of inertia of the system in the untucked configuration are $\dot{\phi}_1$ and J_1, and the moment of inertia in the tucked configuration is J_2, then conservation of angular momentum requires the angular rate in the tucked configuration to be $\dot{\phi}_2 = (J_1/J_2)\dot{\phi}_1$. The planar biped tucks† by shortening its legs to minimum length during the flight phase. To justify the assumption of constant angular rate during the flip, we further assume that the system tucks instantaneously just after the feet leave the ground, liftoff, and that it untucks instantaneously just before the feet touch the ground, touchdown. This simplification results in constant pitch rate during the flight phase of a flip. Later in the paper we relax this assumption by considering the case of slower tucking and untucking.

Frohlich (1979; 1980) points out in his elegant papers on the physics of diving, that a system with several masses can change orientation and angular rate without any angular momentum. This is done by windmilling the arms, peddling the legs, or folding the joints of the body in one sequence and unfolding the body in another sequence. Such configuration effects are not considered here.

5.3.1 Flip Strategies

Equation (5.1) shows a direct trade-off between the pitch rate and vertical rate of the body at liftoff. If the control system increases the vertical rate at liftoff, then a lower pitch rate is needed to rotate the body around in time for landing, and vice versa. The values at liftoff are important because the ballistic nature of the task makes liftoff the last moment the control system can affect either the linear or angular momentum until the next landing. The vertical velocity of the body determines the altitude and the duration of the flight phase, whereas the angular rate determines how far around the body will rotate during that time. The control system must ensure that the system does not over-rotate or under-rotate if it is to continue balanced running. This trade-off between vertical velocity and angular velocity suggests three strategies for producing a flip.

One strategy is to maximize the vertical velocity while adjusting the body pitch rate to provide the correct amount of rotation in the available time. Gymnastics coaches seem to teach this strategy to humans learning the forward flip. The second strategy is to maximize the body pitch rate while adjusting the vertical velocity to produce a flight phase that takes the correct amount of time. The third strategy is to compromise on both angular and vertical rates, perhaps by introducing an additional constraint on the maneuver or an optimization criterion. The control system we implemented uses the first of these three strategies—maximize flight duration and adjust pitch rate accordingly.

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† In describing the actions of the biped running machine we use the terminology of gymnastics. When the biped *tucks* it reduces its moment of inertia by shortening its legs. When it *throws* the body a hip torque is applied that increases the body's rotation rate. In using gymnastic terminology we do not mean to suggest too strong an analogy between the planar biped and a human. The human versions of each of these actions and the human's physical system itself are substantially richer and more elaborate than the planar biped versions we describe here. Moreover, we may find that the suggested functional analogies are not correct.

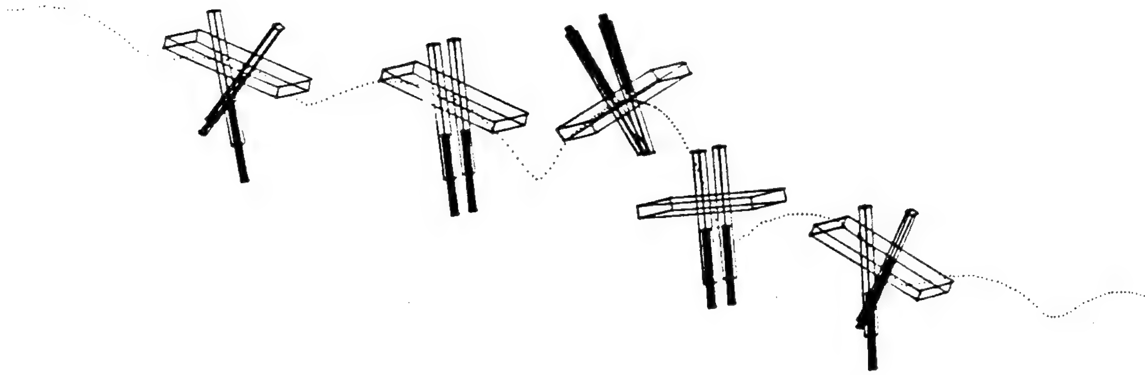

Figure 5–4: Cartoon of planar biped doing a forward flip. The machine was running from right to left. 1) Approach with normal alternating gait, 2) hurdle step to gain altitude and bring the legs together for double support, 3) the body has accelerated forward to initiate the flip and the legs have shortened to increase pitch rate, 4) landing step reduces pitch rate and vertical rate, and 5) resume normal alternating gait. The body configurations are from data recorded from the physical biped during a flip. The dots indicate the path of the center of mass at 12 ms intervals.

5.3.2 Angular Rate During Flip

Because a system doing a flip can have nonzero body pitch angle at both the beginning and end of the flight phase, the total required rotation of the body may deviate from the nominal one revolution that was used in (5.1). When the liftoff and touchdown pitch angles have the right sign—nose down at liftoff and nose up at touchdown—the distance the body must rotate is reduced. Equation (5.1) can be modified to incorporate this reduction in the required rotation angle, $\Delta\phi$. Another correction to (5.1) is required because the legs do not maintain a fixed orientation with respect to the body during flight. At liftoff the body has rotated into a nose down orientation, so the legs are near their extreme forward position with respect to the body. See figure 5–4. During the flip the legs are rotated forward over the top to place them near the back end of their travel. This rotation of the legs and conservation of angular momentum causes backward rotation of the body. We use the notation that when a leg is of length r its moment of inertia is given by $J_l(r)$. If we assume the legs reorient through an angle $\Delta\theta$ with respect to the body, that reorientation takes place when each leg has minimum length r_{min}, and that the body has moment of inertia J_b, then reorientation adds $2\Delta\theta J_l(r_{min})/J_b$ to the required rotation of the body. Modifying (5.1) to account for these factors, the basic flip equation becomes

$$\frac{\Delta\phi_{total}}{2} = n\pi + \frac{\phi_{td} - \phi_{lo}}{2} + \frac{\Delta\theta J_l(r_{min})}{J_b} = \frac{\dot{\phi}\dot{z}}{g}. \tag{5.2}$$

where ϕ_{lo} and ϕ_{td} are the body pitch angles at liftoff and touchdown, assuming positive values for nose down body pitch angle.

To compute the angular rate during the flip, we need to know the angular momentum of the system. The angular momentum of the system is the sum of the angular momenta

of the body and legs. To simplify the analysis we make the approximation that the center of mass of the system remains located at the hip throughout all maneuvers.

The angular momentum of the legs at liftoff is a function of the configuration at liftoff and the forward and vertical speeds. During normal running the net angular momentum of the legs is small because the legs sweep out of phase—one moves forward while the other moves backward. In a flip, however, the legs move together as they sweep backward during stance, giving them substantial angular momentum. The planar biped has telescoping legs as shown in figure 5–3. Calculation of angular momentum for such legs is simple because the orientation and angular rate for all parts of the leg are determined by the hip-foot axis. The angular velocity of the stance leg is

$$\dot{\theta} = \frac{\dot{z}_f x_f - \dot{x}_f z_f}{x_f^2 + z_f^2}, \tag{5.3}$$

where x_f is the forward position of the foot with respect to the center of mass and z_f is the altitude of the foot with respect to the center of mass. The forward and vertical position of the center of mass are x and z. During stance when the foot is stationary on the ground $\dot{x}_f = -\dot{x}$ and $\dot{z}_f = -\dot{z}$. The kinematics of the planar biped are given in figure 5–5. The angular momentum of each leg at liftoff is

$$H_l = \dot{\theta} J_l(r), \tag{5.4}$$

where the moment of inertia of each leg about the hip is

$$J_l(r) = J_{l1} + m_{l1} r_1^2 + J_{l2} + m_{l2}(r - r_2)^2. \tag{5.5}$$

J_{l1}, J_{l2}, m_{l1}, m_{l2}, r_1, and r_2 are physical parameters of the leg and are given in table 5–1. The angular momentum of the body is just $\dot{\phi} J_b$.

The pitch rate of the system once airborne can be found by equating the angular momentum just before liftoff and just after the tuck. If the legs have length r_{lo} just before liftoff, then the angular momentum of the system is

$$\dot{\phi}(J_b + 2J_l(r_{min})) = \dot{\phi}_{lo} J_b + 2\dot{\theta}_{lo} J_l(r_{lo}). \tag{5.6}$$

If the legs shorten immediately after liftoff to length r_{min} and do not swing with respect to the body, then the pitch rate after the tuck is

$$\dot{\phi} = \frac{\dot{\phi}_{lo} J_b + 2\dot{\theta}_{lo} J_l(r_{lo})}{J_b + 2J_l(r_{min})}. \tag{5.7}$$

Equation (5.7) provides a means of predicting the angular rate of the body during the flight phase, given the state of the system just before liftoff.

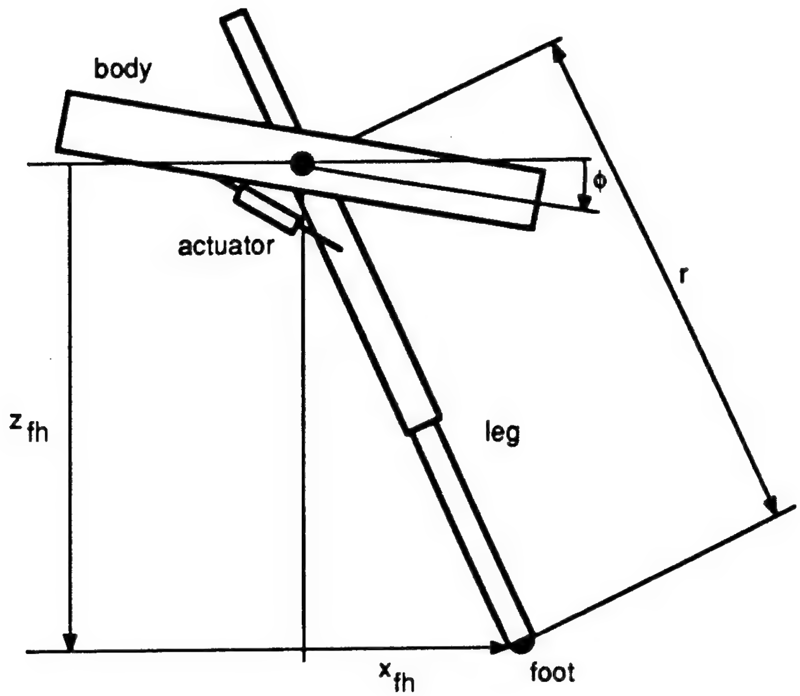

Figure 5–5: Kinematics of planar two-legged running machine. The length of the leg is r, the angle between the leg and vertical is θ and the pitch angle of the body is ϕ. $\theta = \gamma - \phi - 90$. The foot position relative to the hip, $x_{fh} = r \sin\theta$. The kinematics for the second leg are similar except that the hip actuator is attached to the other side of the body.

5.4 Control

To control flips we start with normal biped running and the control algorithms described in Hodgins, Koechling, and Raibert (1985). Briefly, the control system used for normal biped running positions the legs during flight to regulate the forward running speed, thrusts axially with the stance leg to drive the up-and-down bouncing motion of the body, and exerts hip torque between the stance leg and the body to keep the body level. Using these algorithms the machine runs with an alternating gait that uses each leg for support, one at a time, with a flight phase separating each stance phase. The control actions needed for the flip are superimposed upon the normal running behavior produced by this set of control algorithms.

Three steps of the normal running sequence are modified to perform a flip. The three modified steps are the *hurdle step*, *flip step*, and *landing step*. The hurdle step is used to prepare for the maneuver by developing extra hopping height and by making a transition from the normal running gait that uses the legs in alternation, to the double support needed for the flip. The flip step uses both legs together to power the jump, and accelerates the body about the pitch axis for the actual rotating maneuver. The landing step dissipates

Parameter	Symbol	Value
Body mass	m_b	11.45 kg
Body moment of inertia	J_b	0.40 kg-m^2
Upper leg mass	m_{l1}	1.055 kg
Upper leg moment of inertia at COM	J_{l1}	0.0204 kg-m^2
Distance from hip to upper leg COM	r_1	0.0838 m
Lower leg mass	m_{l2}	0.608 kg
Lower leg moment of inertia at COM	J_{l2}	0.0237 kg-m^2
Distance from foot to lower leg COM	r_2	0.317 m
Min leg length	r_{min}	0.44 m
Max leg length	r_{max}	0.67 m
Min leg moment of inertia about hip	$J_l(r_{min})$	0.062 kg-m^2
Max leg moment of inertia about hip	$J_l(r_{max})$	0.126 kg-m^2

Table 5–1: Physical parameters of planar biped.

Step	Action
Approach	Run forward at 2.5 m/s with alternating gait
Hurdle	Hop with maximum thrust Prepare to land on two legs Extend legs further forward than normal
Flip	Jump with maximum thrust Pitch body forward with large hip torque Shorten legs once airborne Lengthen and position both legs for landing
Landing	Hop with small or negative thrust Return pitch rate to zero and restore posture
Following	Resume running with alternating gait

Table 5–2: Summary of actions taken by the planar biped to do a flip.

the high angular and vertical rates and returns the system to the alternating gait. The activities that take place in these three steps are summarized in table 5–2, with additional detail given in Appendix A. We now describe these three steps and how the control system uses them to generate a flip.

5.4.1 Maximum Jump Altitude

Earlier we suggested three possible strategies for establishing the trade-off between pitch velocity and vertical velocity. We decided to control the flip using the first strategy, which attempts to achieve a maximum vertical velocity and an intermediate pitch rate. The rationale for this decision was that it would be easier to remove excess vertical energy with a hard landing than it would be to remove excess angular energy. The large hip torque needed to remove angular energy might demand more traction than would be available, and we were unsure of the ability of the pitch control servo to correct large rate errors.

To get maximum altitude during the flip the control program does three things. It jumps high on the hurdle step to increase the vertical energy in the system, it converts forward speed into vertical speed by placing the foot further forward than normal on the landing just before the flip, and it delivers maximum thrust during the flip step.

The control system delivers maximum thrust to the leg on the hurdle step to increase the altitude that will be reached during the next flight phase. Since the legs are springy, they absorb a portion of the system's vertical energy on landing and then return the absorbed energy to help power the next flight. A hurdle step with increased altitude will result in a flip step with increased altitude as well. Gymnastics coaches for humans generally do not recommend a high hurdle step (George 1980).

Maximum thrust is developed during the hurdle step by setting the hydraulic servovalve that extends the leg to its maximum value as soon as the stance phase begins. On a normal step thrust is delayed to the middle of stance, but thrusting throughout all of stance provides more time for the leg actuator to compress the leg spring and accelerate the body upward.

The second thing the control program does to maximize altitude is to convert some of the forward kinetic energy into vertical kinetic energy. This is done by extending the legs further forward than normal just before the stance phase of the flip step. In normal running, the control system positions the foot to leave the forward and vertical speeds of the body unchanged from one step to the next. The foot position that achieves this result is called the *neutral point* (Raibert 1986a). When the control system places the foot forward of the neutral point, the forward speed declines and the vertical speed increases as shown in figure 5–6.

If the foot were positioned to change the forward speed from $\dot{x}_a$ to $\dot{x}_b$ and if there were no mechanical losses in the leg, then the vertical velocity would increase from $\dot{z}_a$ to $\dot{z}_b = \sqrt{\dot{x}_a^2 - \dot{x}_b^2 + \dot{z}_a^2}$ and the duration of flight would increase by $2(\sqrt{\dot{x}_a^2 - \dot{x}_b^2 + \dot{z}_a^2} - \dot{z}_a)/g$. On a typical flip $\dot{x}_a = 2.5$ m/s and $\dot{x}_b = 1.5$ m/s, so the flight phase could increase by 0.2 seconds if the leg were lossless. We have not measured how much this actually increases flight duration.

The third method of increasing altitude for the flip is to deliver maximum thrust on the flip step itself. During this step there are two legs in support, both of which thrust with maximum hydraulic servovalve settings from the beginning of the stance phase to the end.

Figure 5–7 shows data recorded from the sensors and actuators of the planar biped as

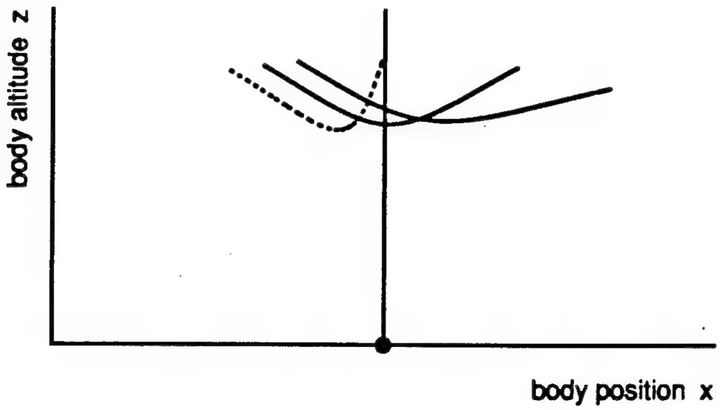

Figure 5–6: Trajectory of body for various foot positions. During normal running the foot is positioned so that the forward velocity of the body is the same at the end of the stance phase as it was at the beginning of the stance phase (solid line). During a flip the foot is extended forward to transfer some of the forward kinetic energy into vertical kinetic energy, thereby increasing the altitude of the flip and the time available to rotate the body (dotted line). It is also possible to convert vertical velocity into forward velocity, a procedure that can be used upon landing after the flip. For each trajectory shown in the plot, the foot is located at the solid circle (•). Adapted from Stentz (1983).

it performed a forward flip. Examining these data, we find that an approach at 2.5 m/s, a hurdle step, and a two-legged jump resulted in a vertical velocity at liftoff of 3.4 m/s, an altitude of 1.04 m, and a flight time of about 0.67 s.

The description so far has centered on maximizing the duration of flight. Assuming this method produces a consistent duration of flight from one flip to the next, the task that remains is to provide one full rotation of the body in the time available. The next section addresses this task.

5.4.2 Desired Body Rotation

We implemented a simple control program for providing the correct amount of pitch rotation. The human operator chooses a running speed which gives the legs a certain angular momentum, the hip actuator throws the body to give it angular momentum, and once airborne the legs shorten into a tuck to increase the angular pitch rate.

The angular momentum of the body is $\dot{\phi}J_b$. The body is given angular momentum by exerting a large nose-down pitch torque about the hip during the final part of the stance phase, just before the flip. This is called throwing the body. When a gymnast throws, he or she typically uses the arms, head, and trunk. The planar biped has no head or arms, so it is restricted to throwing the body. The control system uses two parameters to regulate how much throw the body is given. One parameter is the magnitude of pitch torque. The other parameter is a threshold for the pitch rate—when the pitch rate exceeds this value, the control system turns off the pitch torque. The actual pitch rate exceeds the threshold value by some amount, which we have found to be repeatable.

It is undesirable for the body to over-travel the limited motion of the hip joint during the period of throw. If the body runs out of travel and hits the mechanical stop before liftoff, the collision and resulting ground forces dissipate the angular momentum of the body. To avoid this the control system initiates the throw early enough so that the pitch rate reaches the threshold value at approximately the same time the feet leave the ground at the end of stance. The control system uses a third parameter to specify this delay.

Acceptable values for these three parameters—the delay for initiation of pitch torque, the magnitude of pitch torque, and the threshold pitch rate to terminate pitch torque—were determined empirically through a series of attempted flips. We started with zero delay, maximum pitch torque, and a very large pitch rate threshold. After about 20 attempts with manual adjustment after each, we arrived at values that provided acceptable rotational behavior for a flip.

When the body reaches its peak altitude during the flight phase of the flip, the control system swings the legs part way forward to center the hip joints. As the body approaches the floor, the control system lengthens the legs to untuck for landing and orients the legs to position the feet. The vertical altitude of the body at which the control system begins to lengthen and orient the legs is specified by another parameter, which was adjusted manually throughout the course of attempting several flips. The leg orientation on landing is calculated as in normal running, where the goal is to provide balance and to control forward running speed.

For the flip shown in figure 5–7, the biped approaches with a forward running speed of about 2.5 m/s. After 36 ms of the stance phase of the flip step the control system sets the hip servovalve output signals to 85% of maximum. The hip servovalve is turned off when $\dot{\phi} = 7.85\,\mathrm{rad/s}$. $H_b = 3.9\,\mathrm{kg\text{-}m^2/s}$ and each of the two legs has angular momentum $H_l = 0.42\,\mathrm{kg\text{-}m^2/s}$. The total angular momentum at liftoff is $H = 4.7\,\mathrm{kg\text{-}m^2/s}$. Once the system tucks, the total moment of inertia is $J = 0.52\,\mathrm{kg\text{-}m^2}$ and the rotation rate is $\dot{\phi} = 9.97\,\mathrm{rad/s}$. Equation (5.2) suggests a pitch rate of 9.92 rad/s for the measured values $\phi_{lo} = -.40\,\mathrm{rad}$, $\phi_{td} = .10\,\mathrm{rad}$, and $\Delta\theta = .33\,\mathrm{rad}$.

Once the system lands after the flip, the control system must eliminate the large vertical and angular energies that were needed for the flip. The control system reduces the vertical energy in two ways. It returns the desired forward running speed to the value used before the flip, converting some of the vertical kinetic energy back into forward kinetic energy. This accelerates the system forward while reducing the height of the next hop. The control program also specifies a smaller than usual leg thrust to absorb some of the vertical energy.

To return the body pitch angle and pitch rate to their normal values the control program exerts hip torques between both legs and the body using a linear PD servo:

$$\tau = -k_p(\phi - \phi_d) - k_v(\dot{\phi}), \tag{5.8}$$

where

τ is the hip torque and
ϕ is the pitch angle of the body,
ϕ_d is the desired pitch angle of the body (level),

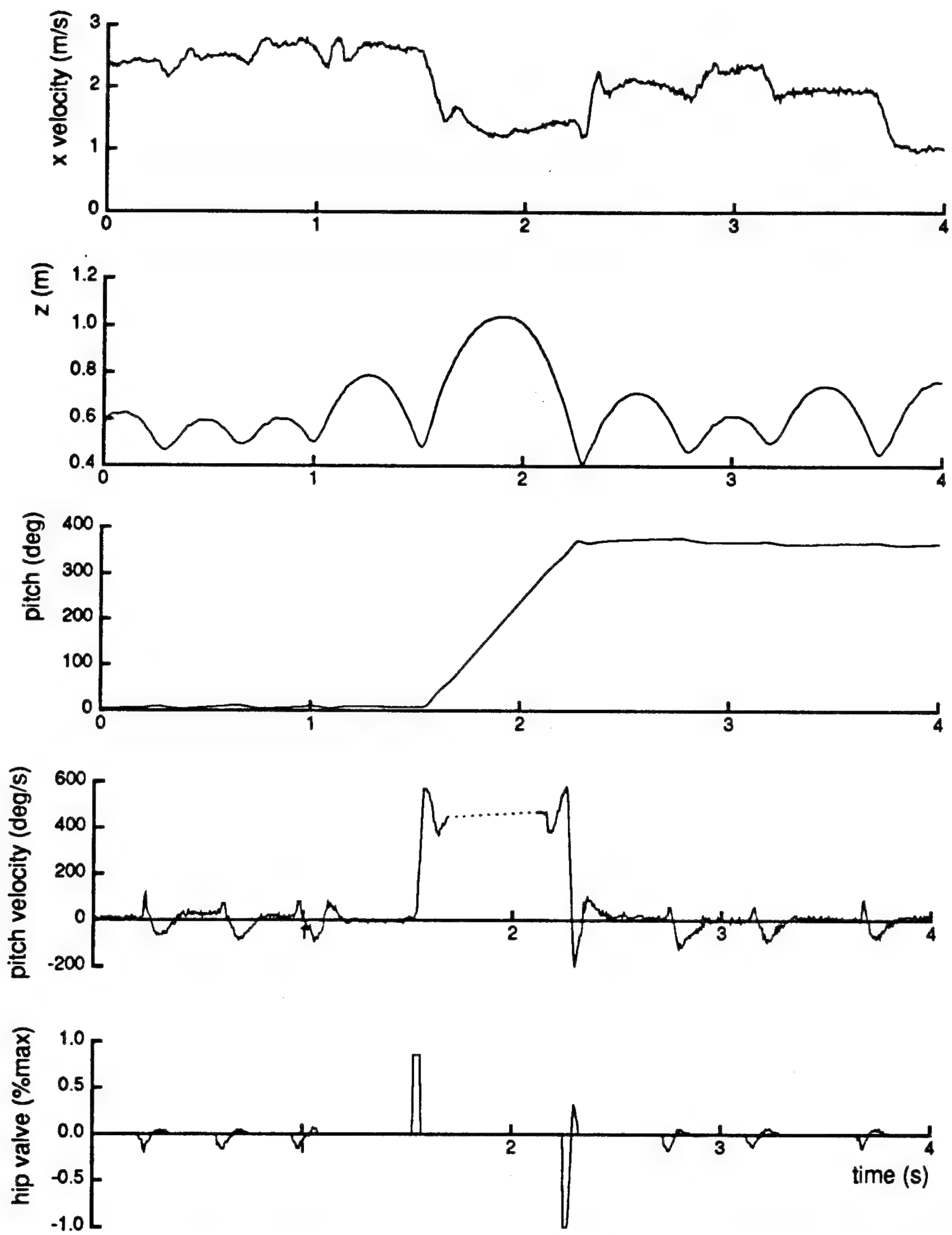

Figure 5–7: Data recorded during a biped flip. The top two curves show that forward speed is converted into vertical motion. The second graph shows that the flip step is the highest in the sequence. The bottom curves of pitch velocity and hip torque illustrate the inherent symmetry in the flip. Data recorded at 6 ms, the cycle time of the control system.

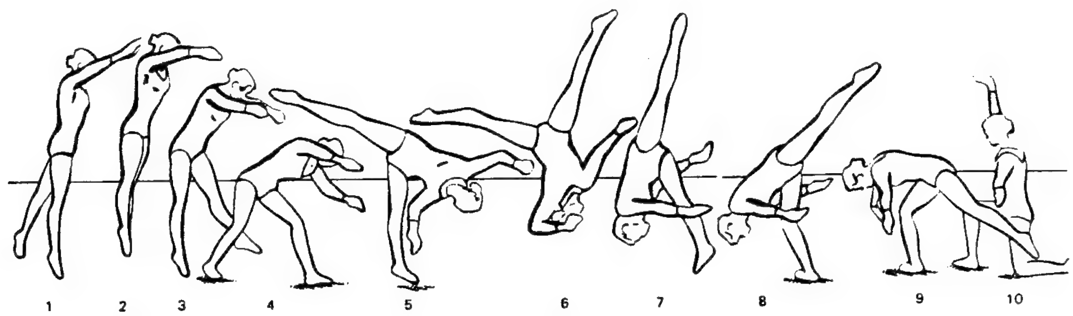

Figure 5–8: Aerial as performed by a human gymnast. Drawings reprinted from Tonry (1983).

$\dot{\phi}$ is the pitch rate of the body, and
k_p, k_v are gains.

This servo is the same mechanism that is used to maintain the body posture in normal running. After the landing step, the control program switches back to an alternating gait with the control algorithms for normal running.

5.4.3 Aerials

The front aerial is a variant of the flip. It differs from the flip in that the performer takes off from one leg rather than two, the legs are spread during the pitch rotation rather than kept together, and the landing takes place on one leg. See figure 5–8. For humans, an aerial is a variant of the cartwheel, in which the hands do not touch the ground. The aerial is considerably easier than a flip for humans, because humans can spread the legs a large amount to reduce the amount of body rotation needed during the flight phase.

For the biped, the aerial is more difficult than the flip. Just one leg is used to power the flight phase, so the duration of flight for an aerial is about 80% of that for a flip, 0.55 s vs. 0.67 s. The reduced flight time makes it difficult to get adequate rotation during the time available in the flight phase. On the other hand, there is no need to swing the legs forward during the flight phase, because the angles between the legs and the body at liftoff are already correct for landing. This reduces the amount the system has to rotate during flight by about 0.10 rad as compared to a flip.

The planar biped control program for aerials differs from that for the flip in only one important characteristic; both the body and the swing leg are thrown to develop angular momentum about the pitch axis. Because the legs move in opposite directions during the approach for an aerial, the net angular momentum of the legs is small. The stance leg sweeps backward while the swing leg sweeps forward. The control program throws the swing leg along with the body to increase the contribution of the legs to the angular momentum of the system. See figure 5–9. The procedure for throwing the body is the same as for the flip, but with just one hip actuator exerting torque. Examining the data shown in figure 5–10, we find that the leg thrown has angular momentum just before liftoff of $H_l = 0.6\,\text{kg-m}^2$.

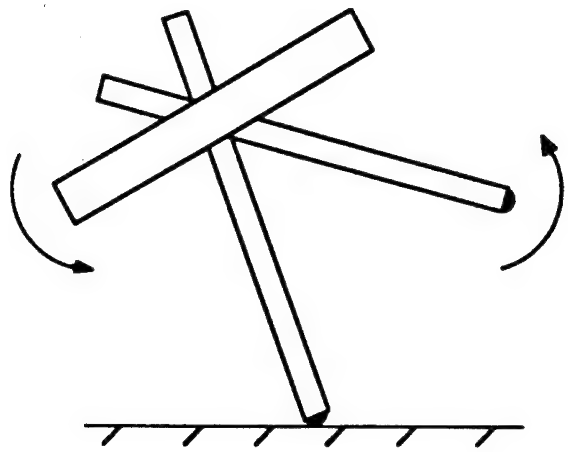

Figure 5–9: To develop angular momentum in the aerial the control system accelerates both the body and the swing leg in the same direction during the final part of the stance phase.

Step	Action
Approach	Run forward at 2.5 m/s with alternating gait
Hurdle	Hop with maximum thrust
	Extend leg further forward than normal
Aerial	Hop with maximum thrust
	Swing free leg backward
	Pitch body forward with large hip torque
	Shorten legs once airborne
	Lengthen and position forward leg for landing
Landing	Hop with small or negative thrust
	Return pitch rate to zero and restore posture
Following	Resume running with normal alternating gait

Table 5–3: Summary of actions taken by the planar biped to do an aerial.

At that time the angular momentum for the stance leg is $H_l = 0.2\,\text{kg-m}^2$, for the body is $H_b = 3.7\,\text{kg-m}^2$, and for the total system is $H_{aerial} = 4.5\,\text{kg-m}^2$. The total is slightly less than for the flip $H_{flip} = 4.7\,\text{kg-m}^2$.

The planar biped executes aerials using the control sequence outlined in table 5–3. Data for one aerial are shown in figure 5–10. The machine has performed aerials successfully many times. In every case, however, the time available for rotation was so short that the control system could not orient the landing leg to properly position the foot for best stability on landing. The system kept its balance on landing, but with a noticeable reduction in forward running speed after the maneuver, as can be seen in the top curve of figure 5–10.

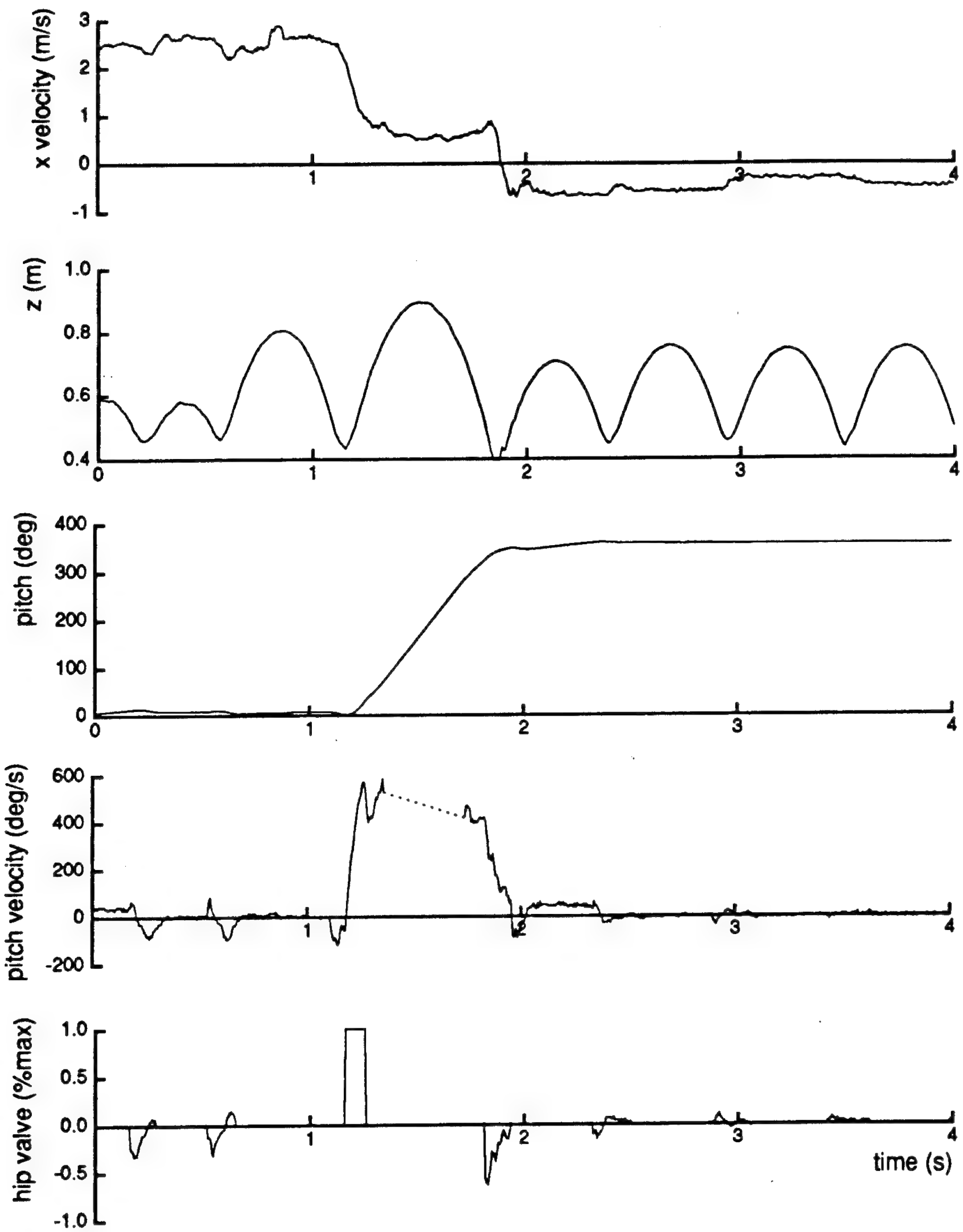

Figure 5–10: Data recorded during a biped aerial. The top curve of x velocity shows how the forward speed is converted to vertical motion, and how after the aerial the foot is not positioned well enough for the machine to continue traveling forward although it does continue running. The second graph shows that the aerial step is the highest in the sequence, although the difference between the hurdle step and the aerial step is not as dramatic as it was for the flip.

5.5 Discussion

5.5.1 Adjusting Pitch Rate During Flight

The control system we implemented for the biped flip makes no attempt to adjust pitch rate once the system is airborne. It keeps the legs tucked during most of the flight phase, resulting in constant moment of inertia and constant angular rate. The amount of rotation during the flight phase is a function of the system's state at liftoff.

It should be possible to control a flip more precisely by manipulating the rate of rotation during the flight phase. Because angular momentum must be conserved during the flight phase, the control system could manipulate the rate of rotation by changing the length and moment of inertia of the legs. To synchronize foot contact with full body rotation, the control system could measure the vertical position and rate of the body and rotational progress of the flip to determine the moment to untuck the legs.

A simplified model assumes that the legs lengthen instantaneously, resulting in just two rates of rotation. If $2J_l(r_{min}) + J_b$ is the moment of inertia of the system with the legs short and $2J_l(r_{max}) + J_b$ is the moment of inertia with the legs long, then to synchronize full rotation with the moment of landing the legs should lengthen when

$$\Delta\phi = \dot{\phi}\left(\frac{2J_l(r_{min}) + J_b}{2J_l(r_{max}) + J_b}\right)\left(\frac{\dot{z} + \sqrt{\dot{z}^2 + 2g(z - z_{td})}}{g}\right) \tag{5.9}$$

where

- $\Delta\phi$ is the remaining rotation required before landing,
- $\dot{\phi}$ is the angular rate of the body with the legs short,
- z is the vertical position of the body,
- z_{td} is the expected vertical position of the body at touch-down, and
- $\dot{z}$ is the vertical velocity.

The control system could evaluate (5.9) throughout the flight phase of the flip to determine when the legs should lengthen to reduce the rotation rate.

Equation (5.9) depends on the ability to lengthen the legs in zero time. For the planar biped, the maximum rate at which the legs can lengthen is determined by the maximum rate at which oil can flow through the hydraulic servovalves, which is a system constant. Equation (5.9) can be modified to accommodate such a fixed rate of leg lengthening.

We start by determining how far the body rotates when the legs are lengthened at a fixed rate. Suppose the legs change length from r_a to r_b at a fixed rate $\dot{r}_k$. Define t_a and t_b so that $r(t_a) = r_a$ and $r(t_b) = r_b$. If the pitch rate is initially $\dot{\phi}_a = \dot{\phi}(t_a)$, then the angular rate during leg lengthening is a function of leg length

$$\dot{\phi}(t) = \dot{\phi}_a\left(\frac{2J_l(r_a) + J_b}{2J_l(r) + J_b}\right). \tag{5.10}$$

Substituting for $r = r_a + (t - t_a)\dot{r}_k$ and integrating we determine how much ϕ changes during lengthening

$$\begin{aligned}\Delta\phi' &= \int_{t_a}^{t_b} \frac{\dot{\phi}_{lo}(2J_l(r_b) + J_b)}{2J_l(r_a + (t - t_a)\dot{r}_k) + J_b} dt \\ &= \frac{a}{\dot{r}_k b}\left\{\arctan\left(\frac{r_b - r_2}{b}\right) - \arctan\left(\frac{r_a - r_2}{b}\right)\right\},\end{aligned} \tag{5.11}$$

where $J_l(r)$ is defined in (5.5), r_2 and m_{l2} are constants defined in table 5–1, and

$$a = -\frac{\dot{\phi}_{lo}(J_b + 2J_l(r_b))}{2m_{l2}}$$

$$b = \sqrt{(r_b - r_2)^2 - \frac{J_b + 2J_l(r_b)}{2m_{l2}}}.$$

We can now solve for the state of the system to begin untucking at constant rate. If the legs are length r_a during the tucked part of a flip, then they should lengthen to r_b at rate $\dot{r}_k$ when

$$\begin{aligned}\Delta\phi = \dot{\phi}\left(\frac{2J_l(r_a) + J_b}{2J_l(r_b) + J_b}\right)\left(\frac{\dot{z} + \sqrt{\dot{z}^2 + 2g(z - z_{td})}}{g} - t_b + t_a\right) \\ + \frac{a}{\dot{r}_k b}\left\{\arctan\left(\frac{r_b - r_2}{b}\right) - \arctan\left(\frac{r_a - r_2}{b}\right)\right\}.\end{aligned} \tag{5.12}$$

5.5.2 Symmetry of the Flip

The locomotion algorithms that are normally used to generate running in the planar biped are based on a principle of control called *running symmetry* (Raibert 1986b). This principle was useful in the design phase of the biped flip. The basic idea of symmetry is that a legged system will travel in steady state when the accelerations it experiences have odd symmetry during each stride. Odd functions integrate to zero over appropriate limits, resulting in no net change in running speed or in posture. For a legged system to run with symmetry throughout a series of steps, the vertical and angular velocities of the body must be coordinated during the flight phases. In normal running the constraint is

$$\frac{\phi}{\dot{\phi}} = \frac{\dot{z}}{g} \tag{5.13}$$

assuming constant angular rate during flight. An implication of (5.13) is that the pitch angle of the body at the end of the flight phase is equal and opposite to the pitch angle

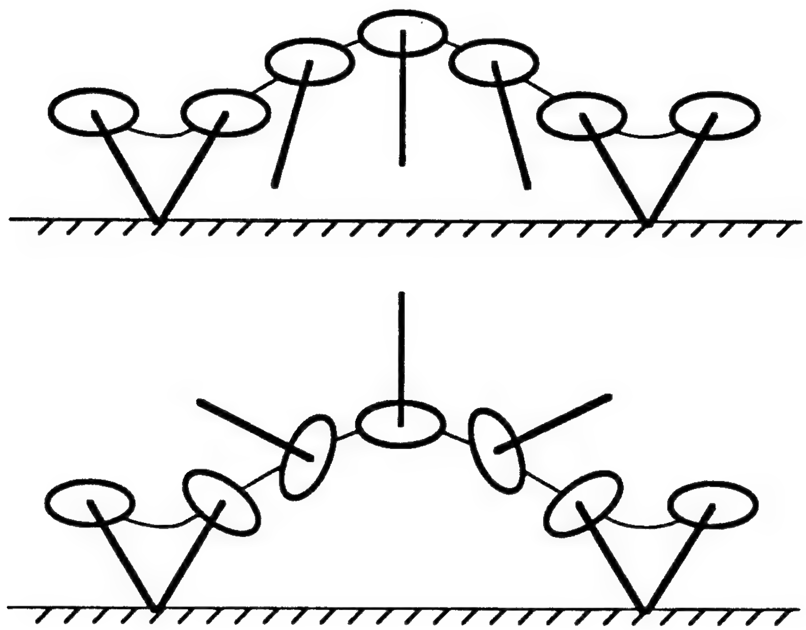

Figure 5–11: Diagram of symmetric behavior during the flight phase for a normal step and for a flip. In both cases the forward displacement and pitch angle of the body are described by odd functions of time, and the altitude of the body is described by an even function of time.

at the beginning of the flight phase $\phi_{td} = -\phi_{lo}$. Despite temporary but radical departures from the steady state, the flip and aerial conform to these symmetries.

For flips and aerials (5.13) becomes

$$\frac{\phi + n\pi}{\dot{\phi}} = \frac{\dot{z}}{g}, \tag{5.14}$$

which is related to (5.1) and (5.2). Equation (5.14) implies the same constraint as does (5.13), but with n additional full rotations of the body during flight. See figure 5–11. Reorientation of the legs with respect to the body during flight is ignored here, but the nonzero pitch angles at liftoff and touchdown are included. The torque exerted at the hip to accelerate the body about the pitch axis to start the flip forms a symmetric pair with the torque exerted at the hip to decelerate the body upon landing—they exhibit the odd actuator symmetry characterized in the theory. The bottom curves in figures 5–7 and 5–10 illustrate this symmetry.

Recognition that flips, too, should conform to symmetry helped us to reason through the design of the flip control programs and to generalize our understanding of symmetry. For instance, at first we worried about getting adequate traction after the flip to dissipate the large angular momentum. Symmetry considerations permitted us to see that if there were adequate traction at liftoff to generate the angular momentum in the first place, then there should be adequate traction at touchdown to dissipate the angular momentum. A

similar consideration was useful in reasoning about the vertical motion.

5.5.3 Open-loop Control

The flip and aerial belong to a class of movements that are dynamic and ballistic. Other members of the class are those that occur in tumbling, diving, and high jumping. These movements are dynamic in that speed and kinetic energy are significant factors in their production. The movements are ballistic in that there are intervals during which the actuators cease to have any direct influence on the key variables of the task. For instance, during the flight phase of a flip no actuator can change the altitude, vertical rate, or angular momentum of the system. In contrast, for the task of making a manipulator move along a trajectory, there is a separate motor to drive the motion of each joint directly and continuously.

Another attribute of these tasks is that outside disturbance plays a relatively minor role. The environment for the movement is essentially unchanging during and between movements, changing only slightly from one repetition to the next. If the initial conditions are precisely established and if the same forces are exerted by each actuator on each repetition, then the behavior repeats in nearly identical form time after time. Sensing and feedback are important in executing such tasks, but in a less direct role than the inner loop of a high bandwidth servomechanism. At the same time, prespecified feedforward control signals can play a more important role in such behavior.

The approach we have taken to control flips and aerials relies on the repeatability of the system, on constancy of the environment, and on repeatably establishing the initial conditions for the movement. These features allow the use of a rather inflexible open-loop actuation pattern to produce the behavior. For instance, to accelerate the pitch rotation of the body the control system waits 36 ms after the start of the flip step, sets the output signal for the hip servovalves to 85% of maximum, waits until the pitch rate reaches 7.85 rad/s, and then turns the hip servovalves off. This control is not devoid of feedback, however, feedback is used sparingly. It uses the pitch rate to determine when to stop throwing the body, and it uses the underlying running pattern to synchronize each flip action to the behavior of the machine.

The control system we implemented for flips is somewhat like a multichannel tape recorder, with the output signal from each channel wired directly to one actuator. To begin a movement the control system establishes the initial conditions for the movement and "it presses the play button" on the tape recorder. In the biological context, this model of motor control has been called the *motor tape* (Evarts et al. 1970). It is thought that under certain circumstances the nervous system may produce patterned behavior, not through sensors and high-gain feedback loops, but by issuing sequences of open-loop commands that go directly to the muscles. In the pure form of the model the commands are issued independent of progress in executing the movement. One can also imagine learning complicated patterns of motion by editing short segments of motor tape together. It is not known whether the motor tape accurately models biological motor mechanisms.

The flip control system is like the motor tape model in that it uses a sequence of motor commands that are issued directly to the actuators, without a local servo. The magnitude of actuator signals for thrust in the hurdle and flip steps and the throw delay and magnitude are specified in this manner. The flip control system is unlike the motor tape in that the timing of control actions are synchronized to events in the locomotion cycle, and thereby, to the progress of the movement. For example, the maximum vertical thrust does not begin until the normal locomotion control algorithm has entered stance. The starting time and magnitude of hip torque for pitch acceleration are determined with a fixed sequence like the motor tape, but hip torque is terminated when pitch rate reaches a desired value.

The use of feedforward without high bandwidth feedback is not new. It is used to send spacecraft to the moon, Mars, and beyond. At a predetermined moment the engines ignite for a predetermined period to take the craft out of Earth orbit and send it toward the remote rendezvous. In space travel there is no attempt to adjust thrust continuously throughout the trip to stay exactly on course with precisely the desired speed, because such an approach would require far too much energy. Occasional adjustments are made instead.

There are two characteristics of space travel that make this approach feasible. First, unexpected external disturbances are minimal. Second, variability in the actuators and internal mechanism is small. These conditions imply that for the same initial conditions and the same actuator output signals, the actual behavior of the system is very nearly the expected behavior and that behavior of the system will reliably repeat on successive executions. These conditions apply to the maneuvers described in this paper.

5.5.4 Where do the Feedforward Control Signals Come From?

Given a task and strategy, the feedforward signal needed to accomplish the task must be found. The feedforward signal can be found using analytical or empirical techniques. The analytical technique is to calculate the feedforward signal from models of the mechanism and the task. For instance, to calculate the feedforward signals for the biped flip described earlier, one might:

1. model the running machine, including its springy legs, hydraulic actuators, and hydraulic power supply, to calculate the maximum attainable vertical velocity of the body.

2. find the appropriate pitch rate using equation (5.2).

3. calculate the forward running speed and the pitch rate of the body required before liftoff using a model of the body and legs.

4. calculate the hip torque needed during stance to develop the desired pitch rate using a model of the body and legs.

5. calculate the output signal that will provide the desired torque using a model of the actuator.

Once again, this approach clearly depends on an a priori specification of the general form of the solution, the strategy. In the case of a flip, the strategy includes the decision to maximize flight time and to adjust body pitch rate accordingly, as well as many other details.

The empirical approach to finding the feedforward signals uses data obtained by attempting to perform the task—learning. Rather than use a model of the system to determine the maximum attainable vertical velocity, the physical system itself is used to make the measurement. Output is applied, the behavior is measured, and the output is adjusted accordingly. The nature of such adjustment and the effectiveness of the results is a long-standing problem.

For both of these approaches, the analytic and the empirical, there is the separate question of what role humans or other outside agents play in the process, as opposed to an automated process. A human could go through the steps described to calculate the control signal from the model, or the control system itself could incorporate these calculations for control. For the empirical approach, a human can look at the results of attempts to do a maneuver and adjust the output for the next try, as we did for the flip and aerial, or the control system could make the adjustment. In the experiments reported in this paper the empirical approach was used and all adjustments were made by humans.

5.5.5 Formulating Strategies

We have described the specific control used to make the biped running machine do forward flips and aerials. The essential features of the approach are a strategy for executing the flip and a set of low-level actions and parameters that implement the strategy. The strategy we chose is based on several decisions:

- Maximize time of flight, with pitch rate adjusted accordingly.
- Extend the legs forward on the flip step to convert forward speed into vertical speed.
- Adjust the start of pitching torque to synchronize the end of hip travel with liftoff.
- Shorten the legs during flight to increase pitch rate.
- Reduce thrust to absorb vertical energy on landing.
- Use normal attitude control algorithm to absorb angular energy on landing.

Each of these decisions was made by humans based on knowledge of the mechanics of the problem and intuition. It is not difficult to imagine that future control systems may be able to formulate strategies such as these automatically. Such systems will embody a model of the mechanical system to be controlled, a working knowledge of the physics that govern behavior of the model, and an ability to reason. Heuristics and optimizations may be important. The need for techniques that bridge the gap between the task level of a motor act and the actuator control level is a deep and important problem.

If strategies for performance of a task were found automatically, optimal behavior might be easier to find. A given strategy for performing a particular task may not be the only

or the best strategy, and it may be difficult to know if a better strategy exists. Richard Fosbury demonstrated this point in 1968 when he introduced a previously unknown form for doing the high jump, the *Fosbury Flop*, in which the jumper goes over face-up rather than face down. Fosbury won an Olympic gold medal in 1968, and Dwight Stones used the Fosbury Flop to set a new high jump world record of $7'7\frac{1}{4}''$ in 1974 (Doherty 1976). Strategy design is crucial for executing maneuvers. Automated techniques may some day permit us to find all possible strategies for a task, and to identify the best possible solution.

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5.7 Appendix A: Details of Control Sequence for Flip

State	Trigger Event	Action	Flip Action
Approach			
LOADING 110	Leg 1 touches ground	Hold leg 1 at landing length Zero hip torque leg 1 Keep leg 2 short Don't move hip 2	
COMPRESSION 120	Leg 1 air spring shortened	Hold leg 1 at landing length Erect body with hip 1 Keep leg 2 short Position leg 2 for landing	
THRUST 130	Leg 1 air spring lengthening	Extend leg 1 Erect body with hip 1 Keep leg 2 short Position leg 2 for landing	
UNLOADING 140	Leg 1 air spring near full length	Shorten leg 1 Zero hip torque leg 1 Keep leg 2 short Position leg 2 for landing	
FLIGHT 150	Leg 1 not touching ground	Shorten leg 1 Don't move hip 1 Lengthen leg 2 for landing Position leg 2 for landing	
Hurdle Step			
LOADING 1210	Leg 2 touches ground	Keep leg 1 short Don't move hip 1 Hold leg 2 at landing length Zero hip torque leg 2	
COMPRESSION 1220	Leg 2 air spring shortened	Keep leg 1 short Position leg 1 for landing † Erect body with hip 2	Reduce desired speed Max thrust: leg 2
THRUST 1230	Leg 2 air spring lengthening	Keep leg 1 short Position leg 1 for landing Erect body with hip 2	Max thrust: leg 2
UNLOADING 1240	Leg 2 air spring near full length	Keep leg 1 short Position leg 1 for landing Shorten leg 2 Zero hip torque leg 2	
FLIGHT 1250	Leg 2 not touching ground	Lengthen both legs for landing Position both legs for landing	

—Continued—

† Expected T_s is adjusted by a factor of $1/\sqrt{2}$ to account for two legs in support during the flip step.

State	Trigger Event	Action	Flip Action
—Continued—			
Flip Step			
LOADING 2110	Both legs touch ground	Hold both legs at landing length	Zero both hip torques
COMPRESSION 2120	Both air springs shortened		Max thrust: both legs Zero both hip torques
THRUST 2130	Compression + delay (36 ms)		Max thrust: both legs Exert large pitch torque: both legs (85% of max)
THRUST 2131	Pitch velocity > desired pitch velocity (7.85 rad/s)		Max thrust: both legs Zero both hip torques
UNLOADING 2140	Both air springs near full length	Shorten both legs Zero both hip torques	
FLIGHT A 2150	Both legs not touching ground		Shorten both legs Don't move either hip
FLIGHT B 2151	Vertical velocity zero		Keep both legs short Center both hips
FLIGHT C 2152	Body altitude < threshold (.7 m)	Lengthen both legs for landing Position both legs for landing †	
Landing Step			
LOADING 3210	Both legs touch ground	Hold both legs at landing length Zero both hip torques	
COMPRESSION 3220	Both air springs shortened	Keep both legs at landing length Erect body: both hips	
THRUST 3230	Both air springs lengthening	Erect body: both hips	Reduced thrust: both legs
UNLOADING 3240	Both air springs near full length	Shorten both legs Zero both hip torques	
Resume normal running			

† Expected T_s is adjusted by a factor of $1/\sqrt{2}$ to account for two legs in support during the flip step.

5.8 Appendix B: Details of Control Sequence for Aerial

State	Trigger Event	Action	Aerial Action
Approach			
LOADING 110	Leg 1 touches ground	Hold leg 1 at landing length Zero hip torque leg 1 Keep leg 2 short Don't move hip 2	
COMPRESSION 120	Leg 1 air spring shortened	Hold leg 1 at landing length Erect body with hip 1 Keep leg 2 short Position leg 2 for landing	
THRUST 130	Leg 1 air spring lengthening	Extend leg 1 Erect body with hip 1 Keep leg 2 short Position leg 2 for landing	
UNLOADING 140	Leg 1 air spring near full length	Shorten leg 1 Zero hip torque leg 1 Keep leg 2 short Position leg 2 for landing	
FLIGHT 150	Leg 1 not touching ground	Shorten leg 1 Don't move hip 1 Lengthen leg 2 for landing Position leg 2 for landing	
Hurdle Step			
LOADING 1210	Leg 2 touches ground	Keep leg 1 short Don't move hip 1 Hold leg 2 at landing length Zero hip torque leg 2	
COMPRESSION 1220	Leg 2 air spring shortened	Keep leg 1 short Position leg 1 for landing Erect body with hip 2	Reduce desired speed Maximum thrust: leg 2
THRUST 1230	Leg 2 air spring lengthening	Keep leg 1 short Position leg 1 for landing Erect body with hip 2	Maximum thrust: leg 2
UNLOADING 1240	Leg 2 air spring near full length	Keep leg 1 short Position leg 1 for landing Shorten leg 2 Zero hip torque leg 2	
FLIGHT 1250	Leg 2 not touching ground	Lengthen both legs for landing Position both legs for landing	

—Continued—

State	Trigger Event	Action	Aerial Action
—Continued—			
Aerial Step			
LOADING 2110	Leg 1 touches ground	Hold leg 1 at landing length Zero hip torque leg 1 Keep leg 2 short Don't move hip 2	
COMPRESSION 2120	Leg 1 air spring shortened	Keep leg 2 short	Maximum thrust: leg 1 Zero hip torque leg 1 Don't move hip 2
THRUST 2130	Compression + delay (36 ms)	Keep leg 2 short	Maximum thrust: leg 1 Exert large pitch torque: leg 1 (85% of maximum) Swing leg 2 backward
THRUST 2131	Pitch velocity > desired pitch velocity (7.85 rad/s)	Keep leg 2 short	Maximum thrust: leg 1 Zero hip torque: leg 1 Swing leg 2 backward
UNLOADING 2140	Leg 1 air spring near full length	Shorten leg 1 Zero hip torque leg 1 Keep leg 2 short	Swing leg 2 backwards
FLIGHT A 2150	Leg 1 not touching ground	Shorten leg 1 Don't move hip 1 Lengthen leg 2 for landing Position leg 2 for landing	
FLIGHT B 2151	Body altitude < threshold (.7 m)	Keep leg 1 short Don't move hip 1 Lengthen leg 2 for landing Position leg 2 for landing	
Landing Step			
LOADING 3210	Leg 2 touches ground	Keep leg 1 short Don't move hip 1 Hold leg 2 at landing length Zero hip torque leg 2	
COMPRESSION 3220	Leg 2 air spring shortened	Keep leg 1 short Position leg 1 for landing Erect body with hip 2 Hold leg 2 at landing length	
THRUST 3230	Leg 2 air spring lengthening	Keep leg 1 short Position leg 1 for landing Erect body with hip 2	Reduced thrust: leg 2
UNLOADING 3240	Leg 2 air spring near full length	Keep leg 1 short Position leg 1 for landing Shorten leg 2 Zero hip torque leg 2	
Resume normal running			

Chapter 6

Quadruped Trotting, Pacing, and Bounding

6.1 Abstract

This paper explores the control of quadruped running gaits that use the legs in pairs: the trot (diagonal pairs), the pace (lateral pairs), and the bound (front and rear pairs). The control algorithms reduce these three gaits to common underlying gait, the virtual one-foot gait. A low-level mechanism reduces the gaits by coordinating the motion of the legs of a pair, and by equalizing the forces the legs of a pair exert on the ground. Once the gaits are reduced, all three can be controlled by a single algorithm, with minor parameter variations. We present experimental data that demonstrate the three running gaits and rudimentary transitions between the gaits. Additional data characterize the energetic cost of locomotion in the quadruped machine, for each of the gaits.

6.2 Introduction

Running is a form of legged locomotion characterized by travel at high speed and by periods of ballistic flight, during which all feet leave the ground. The basic control task is to establish a pattern of leg and body motions that stabilizes the attitude and altitude of the body, while propelling the body in the desired direction at the desired speed (Vukobratović and Frank, 1969). The characteristics of high speed and ballistic flight suggest that dynamics and active stability are important to accomplishing this control task, and to obtaining a better general understanding of running.

We address the task of leg coordination by starting with the algorithms that were used to control one-legged hopping machines, and extending them to control a quadruped running machine. The general goal is to develop a set of locomotion principles that applies

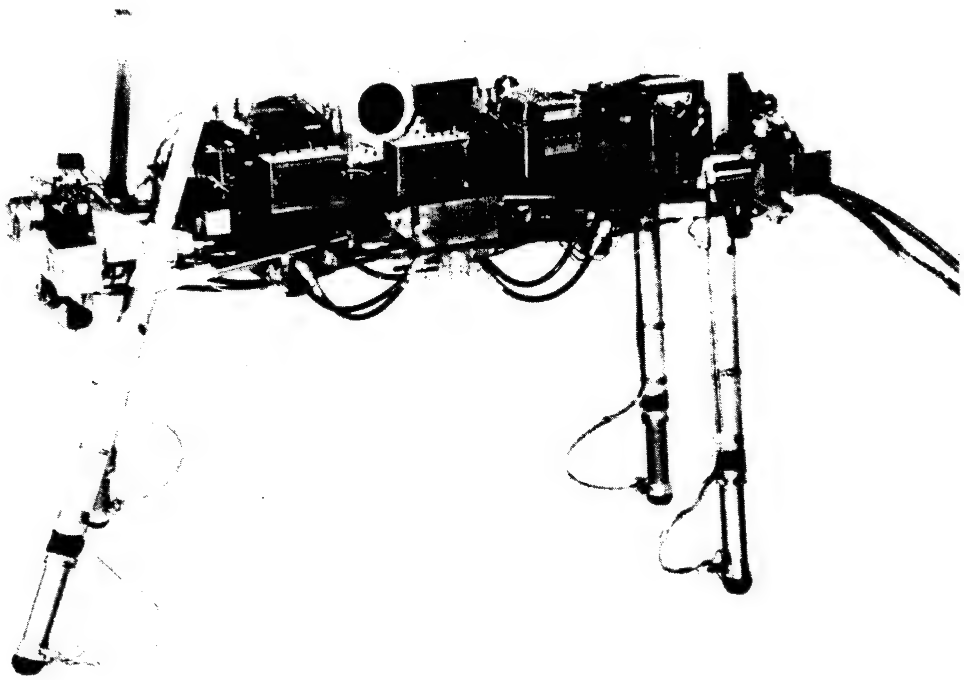

Figure 6–1: Photograph of quadruped running machine used for experiments.

to the dynamic behavior of diverse legged systems, independent of the number of legs.

To develop control algorithms for quadruped running, we start with one-legged hopping, which we have studied in the past(Raibert and Brown 1984, Raibert, Brown and Chepponis 1984). We argue that biped running has the same essential features as one-legged running, and that certain gaits of the quadruped are, in turn, like those of the biped. The first step in the argument is to show that a system with two legs can run as though it had just one. This is possible when the running gait uses just one leg for support at a time, with the other leg shortened and out of the way. The legs alternate like the barrels of a Gatling gun, with only one *firing* at a time. Humans normally run this way. The control algorithms for this mode of running can be like those used to control one-legged machines.

The second step in the argument considers the quadruped gaits that use the legs in pairs: the trot, the pace, and the bound. If the control system can make the two legs that form a pair work together as though they were one leg, then a quadruped can be controlled like a biped. Once this is done, the Gatling approach and the one-legged control algorithms can again be used. The result is a set of locomotion algorithms for controlling systems with several legs, that builds upon the algorithms originally used to control one leg. We have verified the feasibility of this approach with experiments on a four-legged machine that runs

with trotting, pacing, and bounding gaits.

6.3 One-Foot Gaits

Despite several differences†, running on two legs and running on one leg are remarkably similar. For the purpose of this discussion, let us assume that the control algorithms used for hopping machines are adequate for controlling a variety of systems that have a leg, a body, and a suitable collection of actuators and sensors. One can imagine using these algorithms for controlling a biped that runs with an alternating gait, a gait that uses one leg for support at a time. Rather than use one leg for support over and over again, as a hopping machine must, a biped can alternate the use of two separate legs. In human-type biped running, only one of the two legs provides support at a time, only one foot is held in place by friction during support at a time, and only one leg recovers to a forward position during flight. Therefore the thrust each leg delivers during support, the torque generated by each hip during support, and the forward placement of each foot during flight, can each be governed by control algorithms like those used for systems with one leg.

The key characteristics of these running gaits used by one- and two-leg systems are that:

- Exactly one leg provides support at a time.
- Support phase and flight phases proceed in strict alternation.

These characteristics define a class of running gaits that we call the *one-foot gaits*. All the feet remain elevated above the ground, except for the one whose turn it is to provide support. Humans normally run with a one-foot gait, as do one- and two-legged machines we have built.

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† There are several important differences in the running behavior of systems with one and two legs:

- Two legs permit running with substantially reduced pitching of the body. The recovery motion for a one-legged system can occur only during flight, when the angular momentum of the system is fixed. So every recovery motion of the leg must be compensated by a pitching motion of the body. A biped can overlap the backward motion of the supporting leg with the forward motion of the recovery leg, resulting in zero net angular momentum of the legs. The stance leg, whose foot is held in place by friction, can absorb any residual angular momentum by exerting a pitching torque on the body.
- When the stance and recovery motions overlap in time, the duration of flight does not uniquely determine the time available for the recovery motion. Therefore, if the time it takes to recover a leg limits the running speed, then a biped can run faster than a comparable one-legged system.
- If a biped is to recover a leg during stance, then it must be able to shorten that leg during recovery. The recovery leg must be substantially shorter than the support leg if it is to clear the ground without stubbing. Therefore a biped must have a mechanism that will permit the leg to shorten substantially during recovery, and to lengthen again in time for landing. The one-legged systems we explored did not have this requirement, because the recovery motion occurred when the body achieved peak height during flight.
- A biped can generate yaw moments on the body when the legs counteroscillate, swinging fore and aft. This was not a problem with the one-legged systems because the leg swung in the plane containing the center of mass.

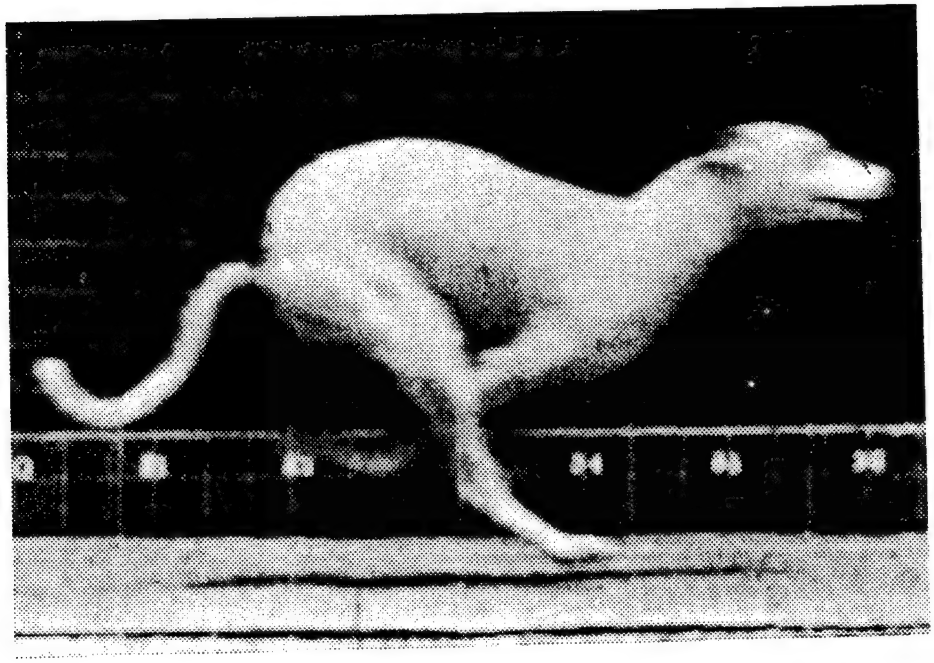


Figure 6–2: Photograph of dog in crossed phase of bound. Flexibility in the back enables the feet to sweep under the center of mass, even with hips located far from the center of mass. Photograph from Muybridge (1957), Plate 121. Reprinted with permission from Dover Press.

The definition of a one-foot gait avoids limiting it to systems with one or two legs. One-foot gaits can, in principle, exist for systems with any number of legs. A quadruped performing a one-foot gait would cycle through the legs, perhaps in a regular order. During the stance phase, a leg would deliver a vertical thrust to maintain the vertical bouncing motion of the body, and its hip would provide a torque to correct the attitude of the body. The next leg would move to a forward position appropriate for landing, some time before it was due to provide support. This pattern of behavior is the Gatling gait we mentioned earlier.

In addition to providing the control functions used for locomotion on one leg, the control system for a quadruped one-foot gait would have to sequence the use of the legs. The sequencing mechanism would select the next leg to provide support so that it could move to a forward position for landing, and it would assign the thrust and attitude control functions to the leg currently providing support. The sequencing mechanism would shorten the other legs to keep them out of the way of the ground until they were needed.

A practical problem with this sort of running is the difficulty of attaching the legs close enough to the center of the body to permit the feet to reach footholds that would provide balance. Each foot must be placed so that the average point of support during the support interval is under the center of mass. It is not difficult to attach one or two legs near the

center of mass, but the design becomes more difficult with more legs. It is also difficult to keep motions of many legs from interfering with one another when the legs are mounted close together. Nature has eased these problems by providing some animals with spines that are flexible enough to permit the feet to reach under the center of mass, even though the hips are located far from the center of mass. For example, see Muybridge's photograph of a running dog, reproduced in figure 6–2. Despite this adaptation, it is not clear that any natural quadruped employs a one-foot gait. Duikers and muntjaks would be the best candidates (Hildebrand 1985).

Even without putting the hips near the center of mass and without a flexible spine, it might be possible for a quadruped to use a one-foot gait if it were not required that each support phase provide perfect balance by itself. The feet might be positioned on either side of the center of mass on alternate steps. Then the system could tip in one direction during one stance phase, and in the opposite direction during the next stance phase. For such a system to balance, the step rate would have to be high compared to the tipping rate.

To summarize this section, the one-foot is a class of running gaits for which only one leg provides support at a time and a flight phase occurs between each stance phase. In principle, one-foot gaits can be defined for systems with any number of legs, but leg interference becomes a practical problem if the hips are all placed near the center of mass, and the ability of the feet to sweep under the center of mass becomes a problem if the hips are moved away from the center of the body. The locomotion algorithms for multi-legged one-foot gaits can be like those used for one-legged systems, provided that the control system also performs sequencing functions that assign the control functions to the legs.

The next step in generalizing from one leg to several legs is to consider pairs of legs that work together. The practical limitations of the quadruped one-foot gaits, namely the difficulty of placing the feet on the neutral point when the hips are separated by feasible distances, can be reduced or eliminated when legs are used in pairs.

6.4 Virtual Legs

Suppose a pair of legs were coordinated to work together, like one equivalent leg. Then a quadruped gait that used the four legs in two pairs can be viewed as an equivalent biped. Quadruped gaits that can be understood and produced in this way are the trot, the pace, and the bound. In each of these gaits the legs operate in pairs. The members of a pair strike the ground in unison and they leave the ground in unison. While one pair of legs provides support, the other pair of legs swings forward in preparation for the next step. Diagonal legs form pairs in the trot, lateral legs form pairs in the pace, and the front legs and rear legs each form a pair in the bound.

An important piece of background comes from Sutherland's work on the construction of a human-carrying walking machine (Sutherland 1983, Sutherland and Ullner 1984). In order to design the hydraulic circuits that would coordinate the load-bearing operation of the machine, Sutherland introduced the idea of a *virtual leg*. He made two or more physical

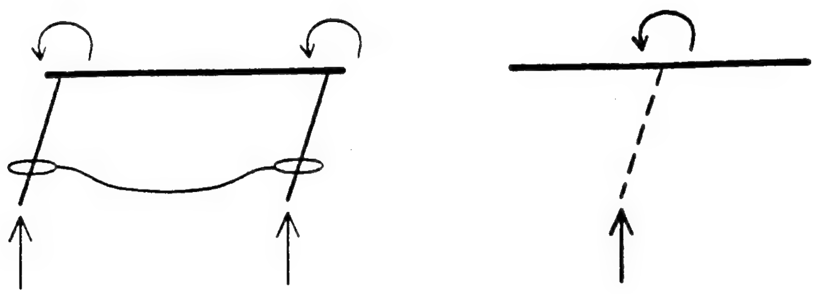


Figure 6–3: Virtual legs. When two legs act in unison, they can be thought of as a functionally equivalent *virtual leg*. The original pair of legs and the virtual leg exert the same forces and moments on the body, so they both result in the same body behavior. A *force-equalizing* virtual leg is shown here. It locates the virtual leg halfway between the two physical legs it represents. Sutherland (1983) introduced the concept of the virtual leg to simplify the design of a six-legged walking machine.

legs act like a single equivalent virtual leg. For instance, when two legs moved downward to lift the machine, each leg supported the same load because a parallel hydraulic circuit equalized the oil pressures. The two physical legs acted like one virtual leg located between them, as shown in figure 6–3. The forces and torques exerted on the body by the set of physical legs and by the virtual leg were equal, so the behavior of the body was the same in both cases. The virtual leg concept permitted Sutherland to express and analyze the complicated behavior of a machine with six legs in simpler terms.

In this paper we use the idea of the virtual leg to separate the problem of generating quadruped running gaits that use pairs of legs in unison, into two simpler problems. One problem is to control the physical legs of a pair, so that the virtual leg they form behaves as desired. A set of operations that coordinate the physical legs to produce desired virtual behavior are

- *Positioning*—Choose positions for the physical feet that place the virtual feet in the desired location.
- *Synchronization*—Ensure that both legs of the pair strike the ground in unison and leave the ground in unison.
- *Force Equalization*—Ensure that both legs of the pair deliver the same axial thrust to the ground.

In order to implement each of these operations, the control system must coordinate the low-level behavior of the physical legs.

The other problem in generating quadruped running gaits that use legs in pairs is to provide locomotion algorithms that specify the desired behavior for the virtual system. Since the trot, the pace, and the bound reduce to virtual biped one-foot gaits, as shown in figure 6–4, the results of the last two sections apply. The locomotion algorithms described earlier for the one-legged systems can be used to specify the behavior of each virtual leg.

To make this approach workable the control system needs a sequencing mechanism that

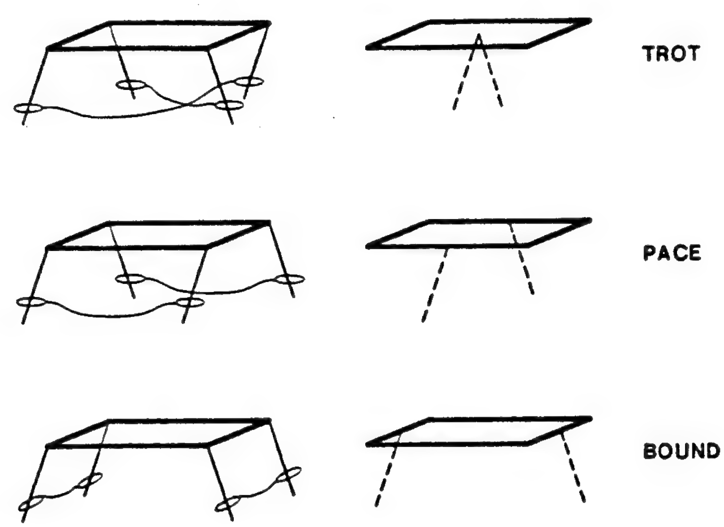


Figure 6–4: Three quadruped gaits use the legs in pairs. In the trot diagonal pairs of legs act in unison, as shown by the shackles. They strike the ground at the same time, they leave the ground at the same time, and they swing forward at the same time. In the pace lateral pairs of legs act in unison. In the bound the front legs act in unison, as do the rear legs. Using the virtual leg idea, each of the gaits shown on the left reduces to the virtual biped one-foot gait shown on the right.

keeps track of the legs and assigns each of the three control functions to the right leg at the right time. The sequencing mechanism would select the next virtual leg to provide support so that the pair of legs could move to a forward position for landing, and it would assign the hopping height and attitude control functions to the virtual leg currently providing support. The control system would also shorten the virtual leg in recovery to keep it from touching the ground while the other virtual leg provides support.

The *force-equalizing* virtual leg we have chosen is a special case of Sutherland's more general concept. When the control system equalizes the axial forces that the legs deliver to the ground, those directed along the main axis of the leg, it moves the effective support point half-way between the physical feet. This choice makes the resulting behavior simple to analyze and similar to that of the one-legged systems we have already studied. It is possible to implement virtual legs that do not equalize axial forces, with correspondingly more complicated behavior. Equations that define the general virtual leg are given in Appendix A. We have considered only the force-equalizing type of virtual leg in this paper.

Relationships between the physical and virtual legs permit the control system to convert the desired behavior of the virtual leg into control signals for each physical leg. For instance, if the physical hips were separated a distance $2d$ in the fore-aft direction, then the control system could add d to the desired virtual foot position to find the desired position for the forward physical foot of the pair, and it could subtract d to find the desired position for the rear physical foot. A similar procedure could position the feet sideways.

Pairs of legs can act together to place the effective point of support close to the center

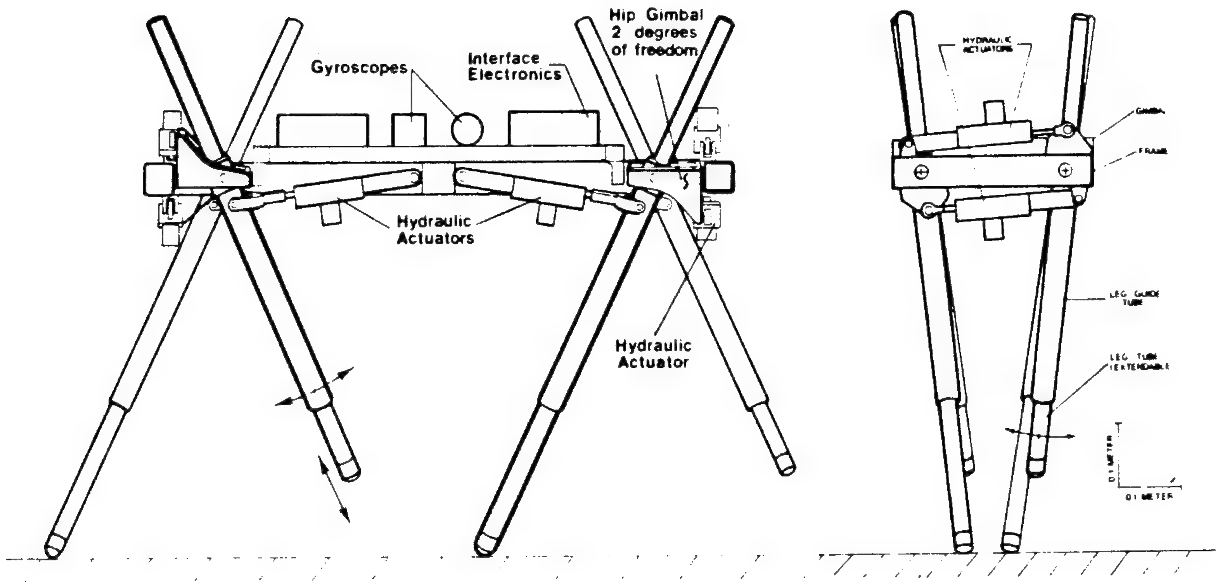


Figure 6–5: Diagram of quadruped running machine used for experiments. The body is an aluminum frame, on which are mounted hip actuators, gyroscopes, and computer interface electronics. Each hip has two low friction hydraulic actuators that position a leg fore and aft, and sideways. An actuator within each leg changes its length, while an air spring makes the leg springy in the axial direction (see figure 6–6). Sensors measure the lengths of the legs, the positions and velocities of the hydraulic hip actuators, pressures in the leg air springs, contact between the feet and the floor, and the pitch and roll angles of the body. An umbilical cable connects the machine to hydraulic, pneumatic, and electrical power supplies, and to the control computer, all of which are located nearby in the laboratory. The arrangement of body, legs, hips, and actuators provides a means to control the position of the feet with respect to the body, to generate an axial thrust with each leg, and to provide hip torques during running. Fore/aft hip spacing is 0.78 m. Lateral hip spacing 0.24 m.

of mass, even though the physical legs are located a substantial distance from the center of the body. The effective point of support provided by a virtual leg lies halfway along the line connecting the two physical feet. The one-foot running gaits described in the previous section would be difficult for a quadruped to use, because it is difficult to locate four hips close enough to the center of mass to position the feet properly, while avoiding leg collisions. In the trot, the gait involving diagonal support pairs, these points are near the center of the body, which we assume is near the center of mass. In the pace, the gait involving lateral support pairs, the lines connecting the feet may pass under the center of mass if the legs are angled inward during stance, or they may pass quite close to the center of mass if the body is narrow.

In the bound, the virtual legs do not provide support under the center of mass. If the body is elongated in the fore-aft direction, then the virtual leg formed by the front legs may not be able to reach under the center of the body. This problem is the same as one described earlier for the one-foot gaits. For this reason it may be incorrect to include the

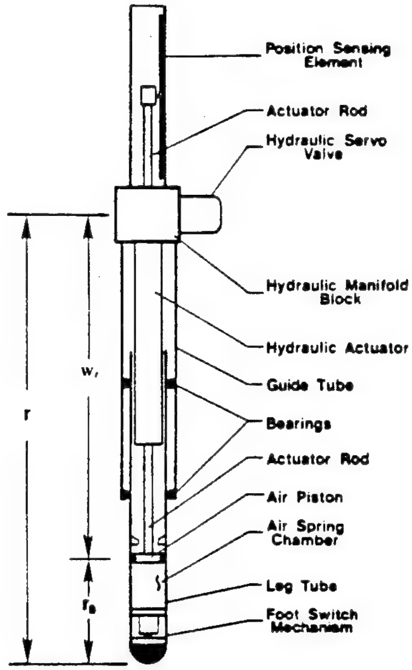


Figure 6–6: Diagram of leg used in running machine. A hydraulic actuator acts in series with an air spring. The hydraulic actuator is used to drive resonant bouncing motion of the machine and to retract the leg during flight. It also acts in conjunction with the air spring to determine the axial force the leg exerts on the ground. Sensors measure the hydraulic length, the overall leg length, the air pressure in the spring, and loading on the foot. Appendix B gives the physical parameters of the machine.

bound in the same class with the trot and pace. Once again, a flexible spine may help to provide a solution, or the control system might be extended to tolerate the pitching motions that occur when the virtual feet do not reach far enough under the body.

To summarize, we use the concept of a virtual leg to separate the task of making a quadruped run into two simpler tasks. One task is to couple the behavior of two physical legs so their combined behavior conforms to the desired behavior of an equivalent virtual leg. The second task is to provide locomotion algorithms for virtual biped one-foot gaits. This approach applies to three quadruped gaits, the trot, the pace, and perhaps the bound.

6.5 Quadruped Running Experiments

In order to explore the use of one-leg algorithms for control systems with several legs, we built and experimented with a four-legged running machine. Figures 6–5 and 6–6 describe the machine.

We designed the control system for these experiments along the lines discussed in the

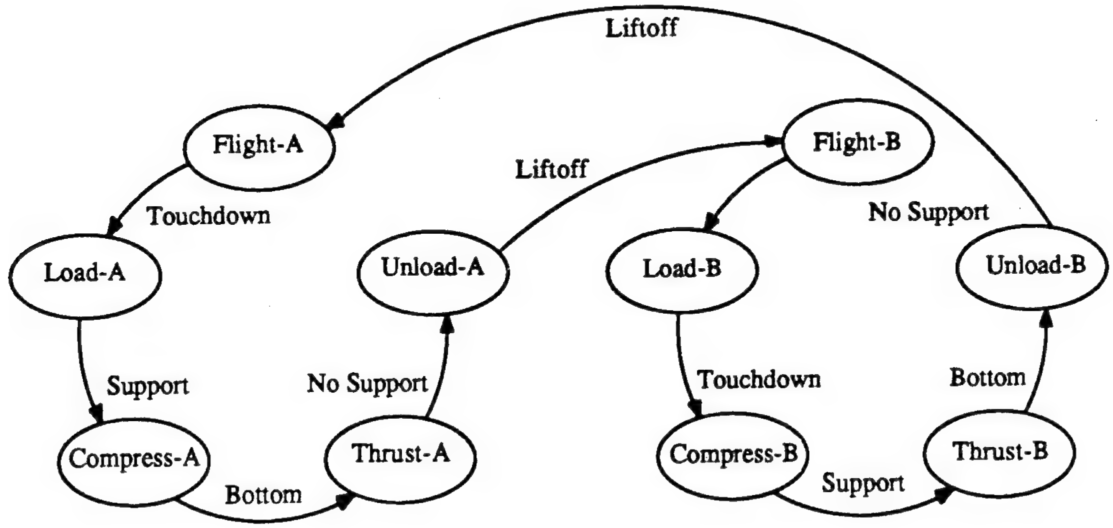


Figure 6–7: Diagram of state machine used to synchronize behavior of quadruped and control programs. Virtual leg $\mathbf{B}_\ell$ swings forward while virtual leg $\mathbf{A}_\ell$ provides support, and vice versa. State transitions are determined by events related to the support leg. See table 6–1 for details.

last two sections. The control system uses the one-leg algorithms to specify desired behavior for each virtual leg, it sequences the virtual legs according to the state in the running cycle, and it coordinates the behavior of pairs of physical legs to act like virtual legs.

6.5.1 Algorithms

The locomotion algorithms previously used to control the one-legged hopping machines were reimplemented to control the virtual legs of the four-legged running machine. With a few exceptions, the same algorithms were used for trotting, pacing, and bounding. The primary variation among the algorithms was the pairing of legs. Other minor differences are discussed below.

To control the forward running velocity, the control system positions the foot of the virtual leg with respect to the center of mass of the body during each flight phase:

$$x_{f,d} = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) \tag{6.1}$$

$$y_{f,d} = \frac{\dot{y}T_s}{2} + k_{\dot{y}}(\dot{y} - \dot{y}_d) \tag{6.2}$$

where

$x_{f,d}$, $y_{f,d}$ is the desired displacement of the foot with respect to the projection of the center of mass,

$\dot{x}$, $\dot{y}$	is the forward running velocity,
$\dot{x}_d$, $\dot{y}_d$	is the desired forward running velocity,
T_s	is the duration of a support period, and
$k_{\dot{x}}$, $k_{\dot{y}}$	are gains.

A human operator uses a two-axis joystick to specify the desired forward running velocity ($\dot{x}_d$, $\dot{y}_d$) during each experiment. The control system estimates the forward velocity of the body, ($\dot{x}$, $\dot{y}$), with the assumption that the feet do not move with respect to the ground during stance. Under this assumption, the backward motion of a foot with respect to the body is equal to the forward motion of the body with respect to the ground. Gyroscope and hip angle measurements are used together with kinematics to make this estimate. The control system assumes that the forward running velocity does not change during flight. The control system measures the duration of stance, T_s, during each stance phase, and it uses the most recent value for control.

To control the pitch and roll attitude of the body during stance, the control system applies torques about the virtual hips, using linear servos:

$$u_x = -k_{p,x}(\phi_P - \phi_{P,d}) - k_{v,x}(\dot{\phi}_P) - k_{f,x}(f_x) \tag{6.3}$$

$$u_y = -k_{p,y}(\phi_R - \phi_{R,d}) - k_{v,y}(\dot{\phi}_R) - k_{f,y}(f_y) \tag{6.4}$$

where

u_x, u_y	are the servovalve output signals for the hip actuators,
ϕ_P, ϕ_R	are the pitch and roll angles of the body,
f_x, f_y	are the forces delivered by the hip actuators, and
k_p, k_v, k_f	are gains.

During fast forward running, the fore-aft hip actuators must move with substantial velocity if the body is to remain level with the foot stationary. An ideal torque control system would generate these high actuator rates without explicitly programming them. In the quadruped we obtained a more nearly ideal response by adding a *sweep* term to the hip actuator output signal. The sweep term adds a component to the servovalve signal proportional to the desired actuator rate, as determined from kinematics, the forward running speed, and the measured pitch rate of the body. The sweep term is added to (6.3).

To control the hopping motion, the control system adjusts the hydraulic length of the virtual leg throughout the running cycle. When a virtual leg is in recovery, the desired hydraulic length is shortened, $w_{\ell,d} = L_1$, to keep the feet from touching the ground. This keeps the virtual leg out of the way. When a virtual leg is preparing for landing or compressing under load of the body, the desired hydraulic length is set to an intermediate value, $w_{\ell,d} = L_2$. During the second part of stance when the virtual leg delivers a thrust to the body, the desired hydraulic length is increased, $w_{\ell,d} = L_3$. The operator specifies L_1, L_2, and L_3 from a control box, with $L_1 < L_2 < L_3$.

State	Trigger Event	Action
1 LOADING $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ touches ground	Equalize axial force $\mathbf{A}_\ell$ Zero hip torque $\mathbf{A}_\ell$ Shorten $\mathbf{B}_\ell$ Don't move hip $\mathbf{B}_\ell$
2 COMPRESSION $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air springs shortened	Equalize axial force $\mathbf{A}_\ell$ Erect body with hip $\mathbf{A}_\ell$ Shorten $\mathbf{B}_\ell$ Position $\mathbf{B}_\ell$ for landing
3 THRUST $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air springs lengthening	Extend $\mathbf{A}_\ell$, equalizing force Erect body with hip $\mathbf{A}_\ell$ Keep $\mathbf{B}_\ell$ short Position $\mathbf{B}_\ell$ for landing
4 UNLOADING $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ air springs near full length	Shorten $\mathbf{A}_\ell$, equalizing force Zero hip torques $\mathbf{A}_\ell$ Keep $\mathbf{B}_\ell$ short Position $\mathbf{B}_\ell$ for landing
5 FLIGHT $\mathbf{A}_\ell$	$\mathbf{A}_\ell$ not touching ground	Shorten $\mathbf{A}_\ell$ Don't move hip $\mathbf{A}_\ell$ Lengthen $\mathbf{B}_\ell$ for landing Position $\mathbf{B}_\ell$ for landing

States 6–10 repeat states 1–5, with $\mathbf{A}_\ell$ and $\mathbf{B}_\ell$ reversed.

Table 6–1: Finite state sequence used for trotting. The state shown in the left column is entered when the event listed in the center column occurs. During normal running states advance sequentially. $\mathbf{A}_\ell$ refers to the virtual leg that uses physical legs 1 and 3 (left front and right rear). $\mathbf{B}_\ell$ refers to the virtual leg that uses physical legs 2 and 4 (left rear and right front). During states 1–5, $\mathbf{A}_\ell$ is the support leg and $\mathbf{B}_\ell$ is the recovery leg. During states 6–10, these roles are reversed.

Sequencing Virtual Legs

Throughout the running cycle, the control system uses a finite state machine to determine which control functions should be applied to which virtual legs. The cycle traverses ten states during each stride. Each state prescribes a set of sensor conditions that triggers transition into the state, and a set of control actions to be taken during the state. The state transitions synchronize the various control functions—vertical thrust, attitude control and foot placement—to the behavior of the running machine. Figure 6–7 and table 6–1 give the sequencing details as they are implemented for the trotting experiments.

Implementation of Virtual Legs

In order to make the legs work together in pairs, the control system coordinates positioning of the physical legs, synchronizes ground contact, and equalizes axial leg thrust. The geometry of the body simplifies the task of positioning the physical legs for trotting. Since the hips are located in symmetric positions about the center of mass, an x-y displacement of both physical feet from the projection of their hips results in a comparable x-y displacement of the virtual foot from the projection of the center of mass. Therefore, the desired position of the virtual foot with respect to the center of mass is used to specify the desired position of the physical legs with respect to their hips:

$$x_{h,i,d} = x_{h,j,d} = x_{f,d} \tag{6.5}$$

$$y_{h,i,d} = y_{h,j,d} = y_{f,d} \tag{6.6}$$

where

$x_{h,i,d}$, $y_{h,i,d}$	is the desired displacement of the ith foot with respect to the projection of the ith hip,
$x_{f,d}$, $y_{f,d}$	is the desired displacement of the virtual foot with respect to the projection of the center of mass,
i, j	designate indices of two physical legs that form one virtual leg.

Once these desired foot displacements are known, transformations based on the kinematics of the legs, hips, and actuators are used to find actuator lengths that will position the foot as desired. These transformations take into account the pitch and roll orientations of the body and the lengths of the legs.

To synchronize the instant of ground contact for the two legs forming a virtual leg, the control system servos the leg lengths during flight, so that both feet have the same altitude. This adjustment affects only the difference in the lengths of the legs, while L_2 determines the average length. Pitch and roll measurements made from onboard gyroscopes and a kinematic calculation are required to perform these adjustments.

This approach to synchronizing ground contact works correctly on a flat floor, but it would fail if used with a support surface that was not flat. An alternative approach that might better tolerate variations in the altitude of the terrain would be to servo the axial leg forces in anticipation of ground contact. When one foot struck the ground, its leg would retract and the other extend. When both feet touch the ground, the stance phase would proceed. This method would rely on fast response from the actuators that retract the legs.

To equalize the axial forces the legs deliver to the ground during stance, the control system servos the differential length of the leg hydraulic actuators:

$$w_{\ell,i,d} = w_{\ell,i} + \frac{r_{s,i} - r_{s,j}}{2} \tag{6.7}$$

where

$w_{\ell,i}$ is the hydraulic length of the ith leg,
$w_{\ell,i,d}$ is the desired hydraulic length of the ith leg, and
$r_{s,i}$ is the air spring length of the ith leg.

This differential adjustment forces the air springs to assume equal lengths and therefore to generate equal axial force. Once again, values for L_2 and L_3 determine the average length of the hydraulic actuators contributing to the virtual leg.

Turning

The algorithms used to control the quadruped, as stated so far, do not control the yaw orientation of the body. We sketch two approaches to controlling yaw that might be accomplished in a quadruped, without interfering with the existing virtual leg control algorithms. One approach manipulates the differential positioning of the feet about the center of mass to achieve a corrective torque about the yaw axis. The virtual leg calculation described earlier specifies the average foot position with respect to the projection of the center of mass. Two degrees of freedom are not specified by these algorithms: the distance between the feet that form the virtual leg and the yaw orientation of the line passing through the feet. The distance between the feet is irrelevant here, but the orientation of the line connecting the feet can be manipulated to generate a yaw torque on the system.

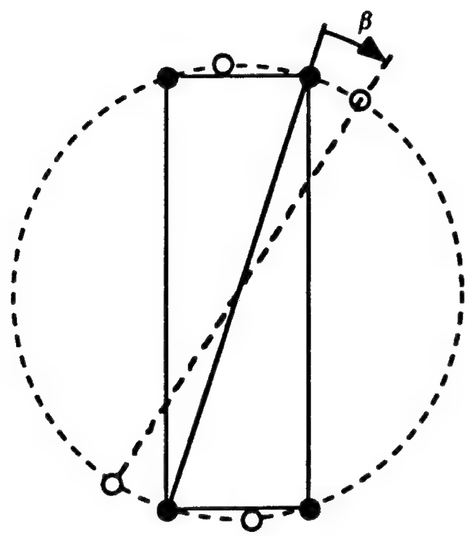


Figure 6–8: Quadruped viewed from above, indicating how the placement of the feet can be used to generate a torque about the yaw axis of the machine. The torque is proportional to the angle between the line through the hips and the line through the feet. The filled circles indicate the location of the hips. The open circles indicate the placement of the feet. β is the angle that determines the turning moment. The foot placement shown in the figure would cause the machine to turn counter-clockwise.

When the system runs in place and each foot is placed directly under its hip, no torque is exerted about the yaw axis. In this case the orientation of the line connecting the feet,

viewed from above, is the same as the orientation of the line connecting the hips. See figure 6–8. When the feet are positioned so the line connecting the feet is rotated about the average foot position, then the axial thrust of each leg has a component about the yaw axis of the system. This component can be used to manipulate the yaw orientation of the quadruped running machine, without disturbing the average position of the feet.

A complementary approach to controlling turning manipulates the pitch and roll hip torques exerted during the stance phase to achieve a corrective torque about the yaw axis. We already indicated that the sum of the pitch and roll torques exerted on the body by the hips is determined by the virtual leg calculation. However, the difference in hip pitch and roll torques can be used to control yaw.

If the hips are separated W laterally, L longitudinally, and the legs are length R, then the yaw torque is

$$\tau_Y = \frac{L}{2R}(\tau_{R,a} - \tau_{R,b}) + \frac{W}{2R}(\tau_{P,a} - \tau_{P,b}). \tag{6.8}$$

Pure yaw torque is obtained without internal forces in the closed chain when

$$\tau_{P,a} = -\tau_{P,b} \tag{6.9}$$

$$\tau_{R,a} = -\tau_{R,b} \tag{6.10}$$

$$W\tau_R = L\tau_P \tag{6.11}$$

where $\tau_{R,a}$ is the roll hip torque exerted by leg a, and $\tau_{P,a}$ is the pitch hip torque exerted by leg a. This manipulation leaves the sum of the pitch hip torques available for controlling the body pitch angle, and the sum of the roll hip torques available for controlling the body roll angle. This method could be used in conjunction with the previous method of influencing yaw described earlier. We have not implemented this method.

To control yaw in the experiments described below, we used the foot placement to generate a yaw moment. The operator specified the desired yaw rate with a lever. The angle between the line connecting the hips and the line connecting the feet, β, is manipulated as follows:

$$\beta = k_{\dot\phi_{Y,d}}\dot\phi_{Y,d} + k_{\dot\phi_Y}\dot\phi_Y \tag{6.12}$$

where

$\dot\phi_Y$	is the yaw angle of the body with respect to room coordinates,
$\dot\phi_{Y,d}$	is the desired yaw angle of the body, and
$k_{\dot\phi_Y}$, $k_{\dot\phi_{Y,d}}$	are gains.

Up to this point we have discussed algorithms for controlling vertical bouncing, forward speed, body attitude, and turning. Once this is done and the desired actuator lengths are known, twelve linear servos act on the hydraulic actuators to position the hips and leg lengths

$$u_i = -k_p(w_i - w_{i,d}) - k_v(\dot w_i) \tag{6.13}$$

where

u_i	is the servovalve output signal for the ith actuator,
w_i, $w_{i,d}$, $\dot{w}_i$	are the position, desired position, and velocity of the ith actuator,
k_p, k_v	are position and velocity gains.

Augmenting (6.1) and (6.2) above to include turning we have:

$$x_{f,d,i} = \frac{\dot{x}T_s}{2} + k_{\dot{x}}(\dot{x} - \dot{x}_d) + D\cos(\beta + \beta_{0,i}) \tag{6.14}$$

$$y_{f,d,i} = \frac{\dot{y}T_s}{2} + k_{\dot{y}}(\dot{y} - \dot{y}_d) + D\sin(\beta + \beta_{0,i}) \tag{6.15}$$

where

i	indicates the physical leg,
β_0, i	is $\arctan(W/L)$ for $i = 1,3$ and $-\arctan(W/L)$ for $i = 2,4$,
D	$= \sqrt{(W^2/2 + L^2/2)}$

6.5.2 Differences in Algorithms for Trotting, Pacing, and Bounding

To a large degree, the same control program is used to produce trotting, pacing, and bounding. The obvious variation is that the legs forming each pair vary among the gaits. However, a few parameters are adjusted differently in producing each gait.

The *track*, the nominal horizontal separation of the feet, is normally set equal to the hip spacing (0.23 m) for trotting and bounding. For pacing, the track is reduced to (0.09 m), to reduce the roll motion of the body. This value of the track parameter brings the feet closer to the midline, which reduces the roll moment during the stance phase.

During the stance phase the legs lengthen to drive the body upward. The amount the legs lengthen is about the same for trotting and pacing, but larger for bounding. This difference is required because the thrust delivered by the legs causes the body to rotate during bounding, and the loading on the legs is reduced. To provide the same vertical acceleration requires longer leg extension.

The most important variation in how the gaits are implemented concerns pitch control during bounding. We found that bounding does not require active control of pitch attitude. The observed pitch oscillation is passively stabilized by the mechanical system. In previous work we found passively stable pitch oscillations occurring in computer simulations of a planar model with two separated legs (Murphy and Raibert 1983). In the bounding experiments reported here the hip actuators are used to position the legs during the flight phase, and to servo the hips to zero force during stance. They do not attempt to control the attitude of the body.

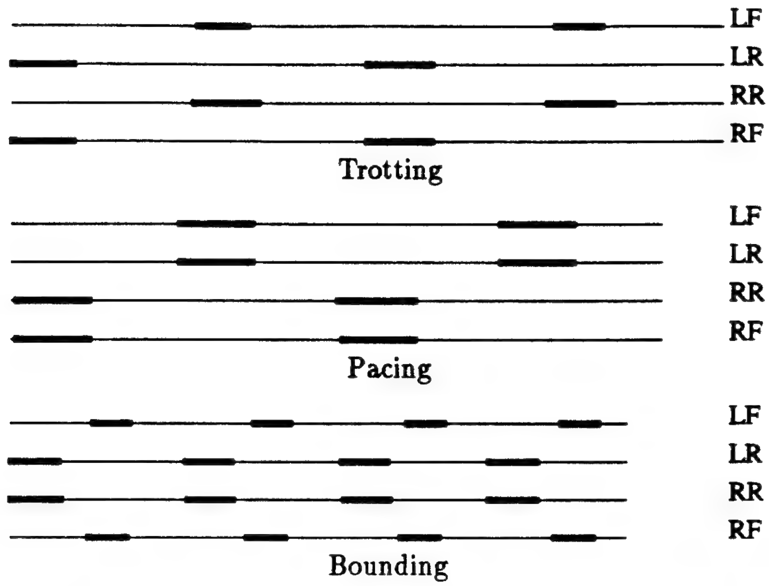


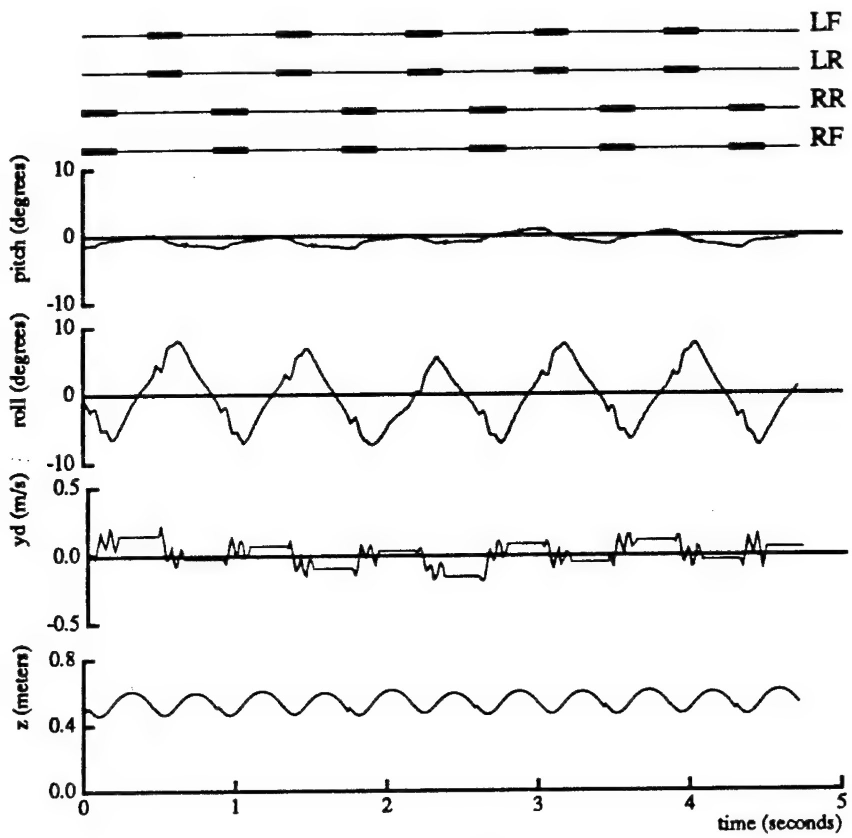
Figure 6–9: Gait diagrams showing the pattern of leg use for the trot, pace, and bound, as executed by the quadruped machine in experiments. The bars indicate periods of ground contact, as measured by load switches in the feet. (Trotting data Q87.335.3, pacing data Q87.142.4, bounding data Q87.196.4.)

Figure 6–10: Quadruped trotting data. These graphs show the bouncing motion that underlies quadruped running. The top curve shows the alternating compression of the air springs for legs 1 and 4. The second curve shows the error in force equalization for the legs that form a virtual leg. One curve (solid) shows the difference in air spring pressures for legs 1 and 3, and the other curve (dash-dots) shows the pressure difference for legs 2 and 4. The bottom curve shows the altitude of the body above the floor, as estimated by the control system. The discontinuities in z are due to errors in estimating the vertical velocity of the body when the feet leave the ground.

Figure 6–11: Forward running. When the joystick specifies a desired forward velocity, the machine accelerates forward. The forward running velocity is not controlled with high precision, as shown by the plots of desired and actual running velocity, $\dot{x}$ and $\dot{x}_d$. The body tips in the direction of running, as shown by the pitch and roll angles. Positive pitch indicates nose down.

6.5.3 Running Data

Data recorded during trotting experiments are shown in figures 6–10 and 6–11. They show that diagonal pairs of legs are used for support in alternation, as required for trotting. The synchronization of foot impacts and equalization of axial leg forces are controlled with good precision as shown by the small differences in axial forces between the legs of a pair.



Pacing

Figure 6–12: Pacing. Data recorded as the quadruped ran in place with a pacing gait. The roll oscillation and lateral translation are characteristic of pacing. Neither trotting nor pacing involve much pitching of the body. For pacing we set the nominal lateral spacing of the feet, the track, to 0.09 m, which is less than the hip spacing, which is 0.229 m. (Data Q87.142.4.)

The vertical bouncing motion of the body is regular and quite smooth.

The control system's ability to regulate forward running velocity is rather poor, as shown in figure 6–11. We observe only a rough proportionality between the desired and actual running velocities. These errors are due to known limitations of the velocity control algorithm. (See Hodgins (1989) for improvements that can be made to the speed control algorithms.) During forward running the inclination of the body about the pitch axis, ϕ_P, deviates from the desired value by as much as 8°. The magnitude and sign of this error are generally related to the forward running velocity. The control system keeps error about the roll axis within about ±5° in these experiments.

Data recorded during pacing are shown in figure 6–12. The behavior is similar to trotting, except for a roll oscillation of about ±5°, and a small amount of lateral motion.

Data for bounding are shown in figure 6–13. In this experiment the machine started bounding in place, accelerated up to 3.0 m/s, then stopped as it approached the end of the running alley. The data show a pitch oscillation of ±18°, with little motion in roll or

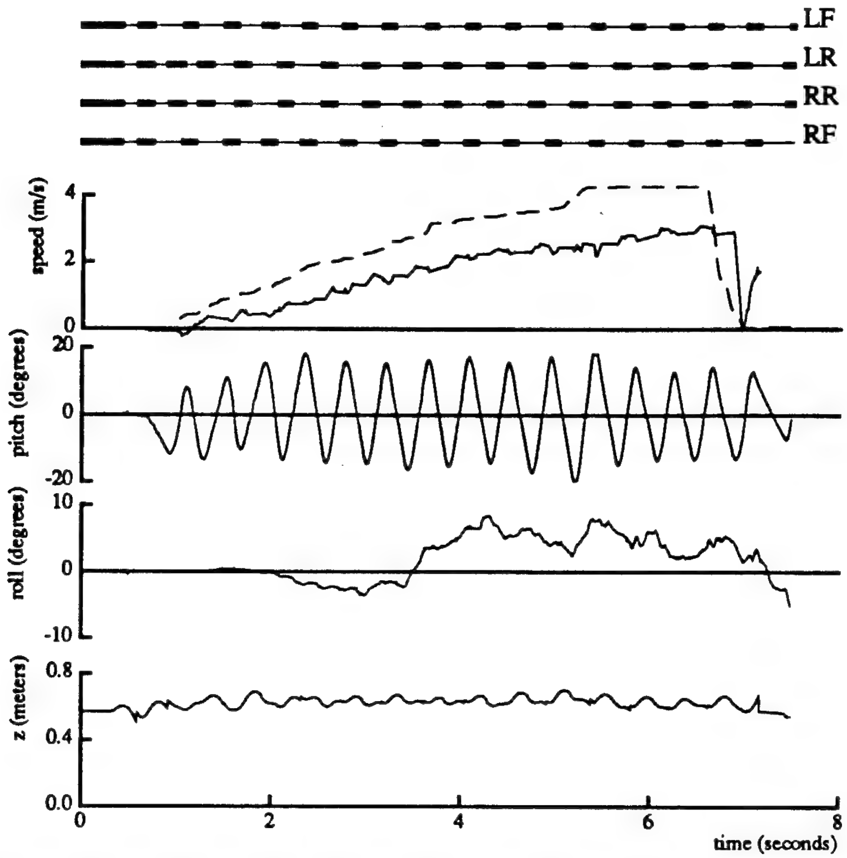


Figure 6–13: Bounding. The machine starts bounding in place, accelerates up to 2.x m/s, and is stopped abruptly at the end of the laboratory. Bounding is characterized by large pitching motions of the body. Vertical motion of the center of mass is smaller than for trotting and pacing. As the machine increases speed, the phase relationship between front and rear ground impacts shifts away from 180°. (Data Q87.196.4.)

yaw. There is an interaction with the phase relationship between front and rear leg support phases and the forward motion of the system. When the quadruped bounds in place, the phase relationship between front and rear legs is stable at 180°. When the quadruped travels forward, the phase shifts so the flight phase occurring after the rear support phase is shorter than the flight phase occurring after the front support phase. This phenomena can be observed in figure 6–13.

6.5.4 Gait Transitions

In previous work we demonstrated that the planar biped could switch between an alternating gait and a hopping gait. The approach was to execute the switch, or gait transition, during the flight phase, when the two gaits are nearly indistinguishable.

The quadruped presents a richer set of gait transition possibilities, as well as more complicated transitions. For instance, there are six different transitions possible among

trotting, pacing, and bounding, and during the flight phase these gaits differ with respect to the characteristic body motion. In trotting, the body is level during the flight phase. In pacing, the pitch angle of the body is level, but the roll angle of the body oscillates. In bounding the roll angle of the body is level, but the pitch angle undergoes oscillations of nearly $\pm 20°$.

One approach to achieving quadruped gait transitions designates a transition step, during which the control system generates the moment required to adjust the attitude motion of the body. For instance, a transition from trotting to bounding would introduce a pitching moment during the transition step, by differentially thrusting with the front and rear support legs. A transition from pacing to trotting would require a moment that eliminated body roll.

So far, we have used this approach on the pitch axis but not the roll axis. Roll axis oscillations are small enough to ignore. We have implemented gait transitions from trotting to pacing, trotting to bounding, and pacing to trotting. Data from these three transitions are shown in figure 6–14. All gait transitions have been done while the quadruped runs in place or travels at low speed.

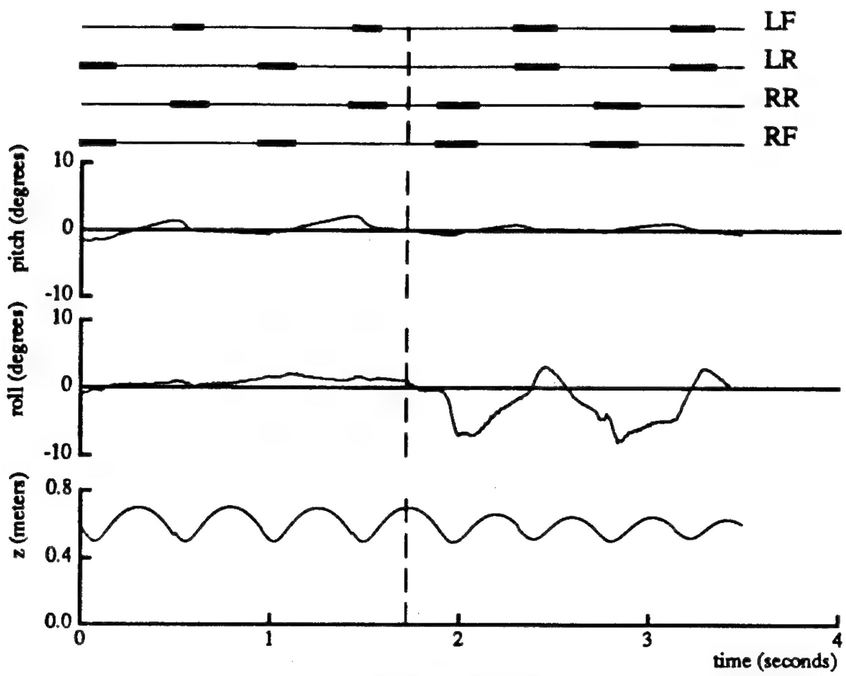
6.5.5 Power Consumption

An important parameter of any vehicle is its power dissipation. We have estimated the hydraulic power dissipated by the quadruped during its various modes of operation. Hydraulic power dissipation can be estimated by taking the product of the oil flow times the system pressure. For the quadruped, the hydraulic pump maintains system pressure constant at 3000 psi. Oil flow is the sum of leakage flows and the volume displaced during actuator motion.

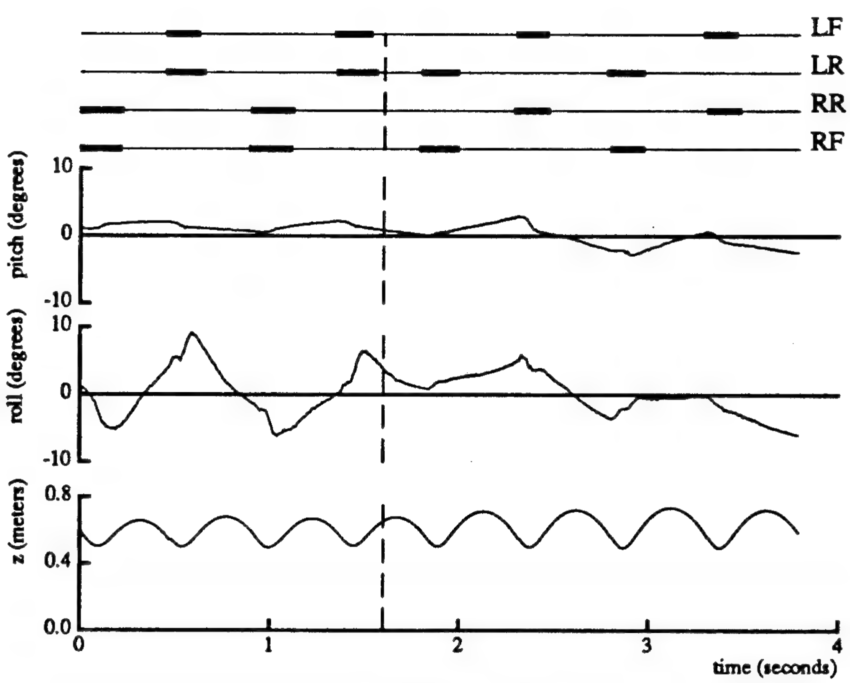
To estimate leakage flow, we measured the total flow through the system with the quadruped standing in place. In this condition, there is leakage flow through the flapper stage of the servovalves and through the clearance seals of the actuator pistons and rods. Leakage flow was measured by running the low pressure return line into a bucket for 30 seconds, and weighing the collected oil. We found that the quadruped consumes about 2 hp in leakage power when standing still.

To estimate flow when running, we integrated the absolute value of each actuator's velocity, and multiplied by the actuator area. The total flow is the sum of flows through all actuators plus the leakage flow. We have assumed that the leakage flow is about constant for standing and running.

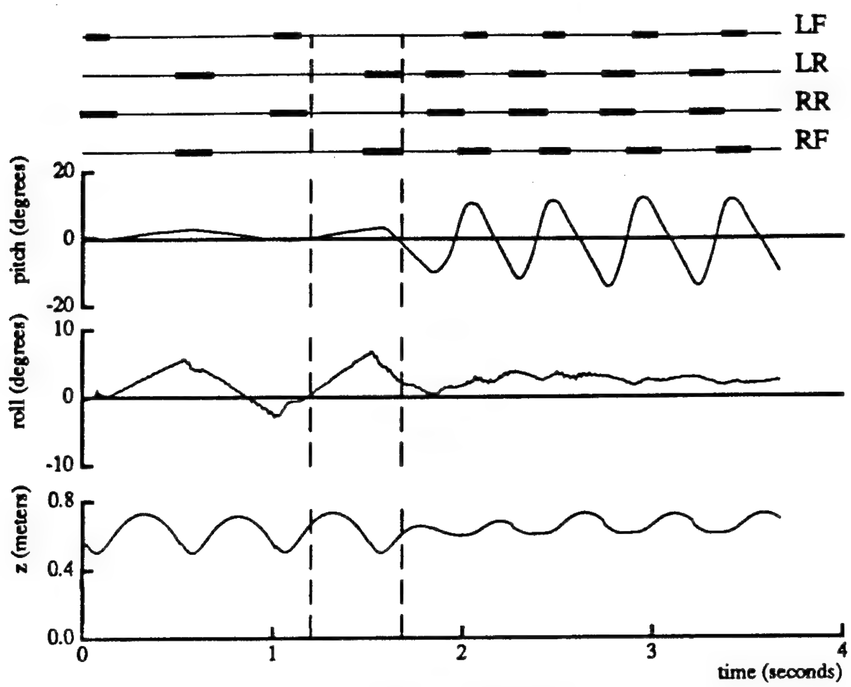
Table 6–2 shows the power dissipation for the quadruped trotting, pacing, and bounding at several speeds. These data show that running speed accounts for about 30% of the total power dissipated, over the speed range of 0 to 2.5 m/s. A large fraction of the power is dissipated in lengthening and shortening the telescoping legs as they go in and out of service. These motions of the legs are essentially unloaded, and could be achieved at much lower power cost with a different leg mechanism design. The quadruped running machine was not designed with efficiency in mind.



Trot to Pace



Pace to Trot



Trot to Bound

Figure 6–14: Gait transitions. Data recorded during gait transitions: Opposite Top) trot to pace Opposite Bottom) pace to trot, and Above) trot to bound. Transitions between trotting and pacing occur smoothly, without much disruption. It take several steps before a transition from trotting to bounding stabilizes. All transitions shown are for running in place. (Trot to pace Q87.335.3; pace to trot Q87.335.4; trot to bound, Q88.2.3.)

Gait	Power Dissipation (hp) 0 m/s	1 m/s	2 m/s	2.5 m/s
Trot	3.25	3.75	4	4.5
Pace	3.4	3.5	-	-
Bound	3.75	3.85	4	4.2

Table 6–2. Power dissipation in the quadruped during standing, trotting, pacing, and bounding. Each figure includes the 2 hp leakage loss measured in standing.

6.6 Discussion

We have limited this paper to study of force-equalizing virtual legs. The force-equalizing virtual leg has the virtue of keeping the effective point of support, the virtual foot, located halfway between the physical points of support. This makes the task of positioning the point of support comparable to the one-legged system's task of positioning the foot.

This similarity permits us to bring our experience with one-legged systems to bear on the quadruped control problem.

A consequence of using the force-equalizing virtual leg, however, is to limit the control system's performance. In restricting the control system to this special case, we have given up a degree of freedom in the control. Without this restriction, the control system could adjust the differential axial leg force and the hip torque to manipulate the location of the virtual foot *during* the support interval. Such adjustments could correct the forward running velocity with finer temporal resolution than the once-per-step method described in this paper. However, manipulating the location of the virtual foot during support would increase the complexity of the control. Such additional control might be particularly useful at low stepping rates, or for systems that tip over quickly because of short legs.

Another consequence of using force-equalizing virtual legs is the loss of passive stability that a set of legs might otherwise provide. A table resists tipping when unevenly loaded, because the legs near the load generate more force than the legs that are far from the load. If a table had force-equalizing legs, then an uneven load would cause the legs near the load to shorten, the legs remote from the load to lengthen, and the surface to tip. This force-equalized behavior should be expected, since it is precisely the behavior of a table with just one leg located in the middle. This one-legged behavior is what we set out to accomplish in the first place.

Our experiments have shown that this approach which discards passive stability of the legs is workable, but it leaves us in a philosophical quandary. On the one hand, the force-equalizing virtual leg permits relatively sophisticated behavior with an extremely simple implementation, largely because it permits us to build on previous results. On the other hand, we believe that a well-engineered control system should take advantage of the intrinsic mechanical properties of the mechanism. If the machine is cleverly designed its intrinsic mechanical behavior will be the desired behavior. The control system need only to fine tune this correct system behavior, not fight the mechanism to make it obey. Such an approach that splits responsibility for good behavior between the mechanical design and the control design leads to simpler, hardier, and more efficient machines. Because the control system that uses the force-equalizing virtual leg discards the passive stability available from the legs rather than somehow harnessing it, we expect that it will eventually be replaced by a better method.

Despite these limitations, it is entirely possible that four-legged animals use force equalization when they trot, pace, or bound. One might find out by measuring the axial forces that develop in the legs of running quadrupeds, perhaps using sets of force platforms. The experiment would disturb one of the feet during stance by shifting the support surface upward or downward. If force equalization were in effect, the difference in axial leg force would not be affected by the manipulation. Exact force equalization is unlikely to be found, because the distribution of mass in animals' bodies is skewed by the asymmetric placement of their heads and often by unequal lengths of the fore and hind legs. One might expect, therefore, to find that the forces delivered by the legs vary in proportion to the loading caused by distribution of mass in the body.

So far, we have considered virtual legs that represent the behavior of physical legs acting in unison. A plausible extension of the concept might represent the behavior of physical legs that act in sequence, possibly overlapped in time. The simplest approach would separate each support interval into subintervals, during which a fixed set of legs would provide support. Then the entire support interval might be represented by a sequence of virtual support phases. For a rotary gallop the sequence of phases would be (1) right rear, (2) right rear and left rear, (3) left rear, (4) left rear and left front, (5) left front, (6) left front and right front, and (7) right front. A key challenge in this problem is to find a mechanism that can mediate the smooth exchange of support from one leg to another without disrupting the bouncing motion of the body. With such an approach one might understand and produce galloping with control techniques no more complicated than those described in this paper.

This paper discusses methods for generating several gaits, but it is silent on the issue of choosing which gait to use. In animals, energetic cost seems to be an important factor in selecting a gait. Animals change gait as they change speed in order to minimize the cost of transportation. The geometry of the animal may also enter into gait selection. At low running speeds, for instance, long-legged animals use a pace rather than a trot, presumably to avoid interference between the front and rear legs on each side. Other factors, such as the range of leg motion and stiffness, may also be important. Despite these potential factors, we do not yet have any clear criteria for selecting one gait over another.

6.7 Summary

- Previous work resulted in locomotion algorithms that were effective for controlling systems with a body and one springy leg. The control system was decomposed into three parts: hopping, body attitude, and forward velocity.
- There is a class of gaits called the one-foot, for which only one foot touches the ground at a time. In principle, the locomotion algorithms that were effective for one-legged machines could be used to control a variety of systems executing one-foot gaits, independent of the number of legs. Control systems for N-legged one-foot gaits would also need to sequence the legs in their use.
- The behavior of a pair of legs that act in unison can be represented by an equivalent virtual leg. Virtual legs are used to map several quadruped running gaits—the trot, the pace, and the bound—into virtual biped one-foot gaits. This approach to running decomposes the control problem into a part that uses the behavior of the virtual legs to control the body, and a part that coordinates the actions of the legs.
- Experiments with a four-legged running machine verify the general approach outlined in this paper. The control system uses the one-legged algorithms, a finite state machine, and virtual legs to make it run with trotting, pacing and bounding gaits.
- Gait transitions from trotting to pacing and from pacing to trotting were accomplished at low forward speed by switching from one gait to the other during the flight phase.

Transitions from trotting to bounding were accomplished by interpolating an adjustment step, during which differential thrust of the fore and aft legs gave the body a pitch moment.

- Power dissipation measurements show that the quadruped dissipates about 2 hp to stand still, about 3.25 hp to hop in place, and about 4.2 hp to run with a speed of 2.5 m/s.

6.8 References

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6.9 Appendix A: Equations for Virtual Leg in the Plane

This appendix describes the relationships between a general planar virtual leg and the two physical legs it represents. The equations given below were developed to express the angular position θ, axial force f, and hip torque τ for the virtual leg in terms of variables that describe the behavior of the two physical legs. The analysis is for the static case. The configuration and variables are defined in figure 6–15.

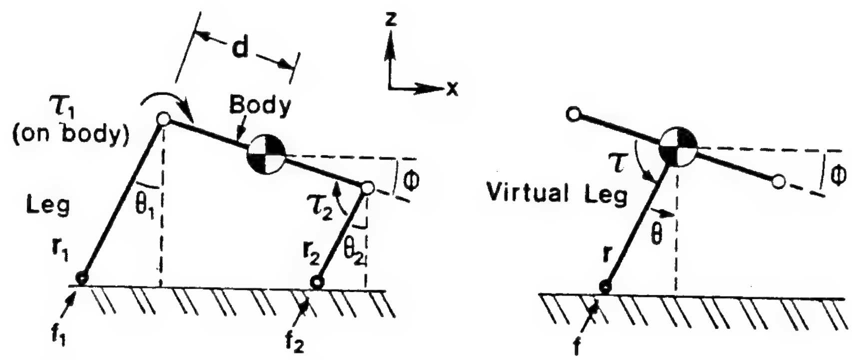

Figure 6–15: Model used for analysis of virtual leg.

We assume that the virtual foot is located on a line connecting the physical feet. This yields the geometric constraint

$$r_1 \cos\theta_1 + r_2 \cos\theta_2 = 2r\cos\theta = 2A. \tag{6.16}$$

Body angle ϕ is not independent of θ_1, θ_2, r_1, r_2, and d, so

$$r_1 \cos\theta_1 - r_2\cos\theta_2 = 2d\sin\phi. \tag{6.17}$$

By summing forces and moments we obtain

$$\begin{aligned}\sum F_x &= f_1 \sin\theta_1 + f_2\sin\theta_2 - \frac{\tau_1}{r_1}\cos\theta_1 - \frac{\tau_2}{r_2}\cos\theta_2 \\ &= f\sin\theta - \frac{\tau}{r}\cos\theta \\ &= B,\end{aligned} \tag{6.18}$$

$$\begin{aligned}\sum F_z &= f_1\cos\theta_1 + f_2\cos\theta_2 + \frac{\tau_1}{r_1}\sin\theta_1 + \frac{\tau_2}{r_2}\sin\theta_2 - mg \\ &= f\cos\theta - \frac{\tau}{r}\sin\theta - mg \\ &= C,\end{aligned} \tag{6.19}$$

$$\begin{aligned}\sum M_{cg} &= -f_1 d\cos(\theta_1 - \phi) + f_2 d\cos(\theta_2 - \phi) \\ &\quad - \frac{\tau_1}{r_1}(r_1 + d\sin(\theta_1 - \phi)) - \frac{\tau_2}{r_2}(r_2 - d\sin(\theta_2 - \phi)) \\ &= -\tau \\ &= -D.\end{aligned} \tag{6.20}$$

Using the lumped variables A, B, C, and D, we can solve (6.16)–(6.20) for θ, r, τ, and f:

$$r = \frac{A}{\cos\theta}, \tag{6.21}$$

$$\theta = \arctan\left(\frac{B + D/A}{C}\right), \tag{6.22}$$

$$\tau = D, \tag{6.23}$$

$$f = \frac{B + (\tau/r)\cos\theta}{\sin\theta}. \tag{6.24}$$

How do these results square with the force-equalizing virtual leg described in the text? For $f_1 = f_2$, $r_1 \sin\theta_1 = r_2 \sin\theta_2$, and small ϕ,

$$r \approx \frac{r_1 + r_2}{2}, \tag{6.25}$$

$$\theta \approx \frac{\theta_1 + \theta_2}{2}, \tag{6.26}$$

$$f \approx 2f_1, \tag{6.27}$$

$$\tau \approx \tau_1 + \tau_2. \tag{6.28}$$

6.10 Appendix B: Physical Parameters of Four-Legged Running Machine

Parameter	Metric Units	English Units
Overall length	1.05 m	41.2 in
Overall height	0.95 m	37.5 in
Overall width	0.35 m	13.8 in
Hip height (max)	0.668 m	26.31 in
Hip spacing (x)†	0.776 m	30.56 in
Hip spacing (y)	0.239 m	9.40 in
Leg sweep angle (x)	± 0.565 rad	±32.4°
Leg sweep angle (y)	± 0.384 rad	±22.0°
Leg stroke (hydraulic)	0.229 m	9.0 in
Leg stroke (spring)	0.102 m	4.0 in
Body mass	25.2 kg	55.4 lb
Body moment of inertia (x)	0.257 kg-m^2	880 lb-in^2
Body moment of inertia (y)	1.60 kg-m^2	5470 lb-in^2
Body moment of inertia (z)	0.86 kg-m^2	6340 lb-in^2
Leg mass, total each	1.40 kg	3.08 lb
Leg mass, unsprung	0.286 kg	0.63 lbm
Leg moment of inertia (about hip)	0.14 kg-m^2	480 lb-in^2
Leg spring stiffness @20 psi (fully extended)	2100 N/m	12 lbf/in
Hip torque, @2000 psi (x)	111 N-m	983 in-lbf
Hip torque, @2000 psi (y)	77.6 N-m	687 in-lbf
Leg thrust, @2000 psi	765 N	172 lbf

† x—fore and aft, y—sideways, z—up and down.

Chapter 7

Articulated Leg

7.1 Abstract

Central to the design of a legged system is the mechanical design of the leg itself. Legs are the elements that exert forces on the body to propel the body forward for transport, to counteract gravitational loading, and to keep the body in an upright posture. Most legs designed for legged machines are intended to be rigid, yet animals have legs that deform substantially under load. Compliance in legs can improve efficiency, reduce maximum loading, and simplify control. We have built and tested a number of leg designs on machines having one, two and four legs. These designs use linear telescoping joints to change length and gas springs for axial compliance. This paper discusses a leg design that uses a rotary "ankle"joint for control of length and a fiberglass leafspring as the compliant element. We expect an articulated leg design to yield better performance, reliability, and simplicity. In this chapter, the pros and cons of the telescoping leg and the Monopod's present articulated leg are studied and an alternative improved leg design is considered. In addition, we discuss the design of a mechanism that constraints the motion of experimental legged systems to the plane.

7.2 Introduction

An intriguing characteristic of legs found in nature is their ability to deform elastically during running (Cavagna 1970). The elements primarily responsible for elastic deformation in animal legs are the muscles, tendons, and ligaments. Elastic deformation is used by biological systems to recover a portion of the energy expended during a stride, and to return that energy on the next stride. This can reduce the total cost of transport (Dawson and Taylor 1973; Alexander and Vernon 1975; Cavagna et al. 1977; McMahon 1984). Another function of compliance in biological legs is to reduce the impact forces and peak loads that

are experienced by the leg, the body, and the support surface. Finally, there is the possibility that the compliant character of biological limbs can simplify the control task performed by the nervous system.

In contrast to the compliance of legs found in nature, most legged robots have legs that are designed to be stiff. Because legged machines typically walk rather than run, there may be less need for compliant legs. For instance, locomotion can be energy efficient if the legs move the body in a purely horizontal motion with no actuators absorbing energy. Several legs have been designed according to this principle (Lucas 1894; Hirose and Umetani 1980; Waldron and Kinzel 1983). Impact forces may be kept small during walking by bringing each foot into contact with the ground at low relative speed. Small leg mass and low speed reduce the cost of accelerating the swing motion of the leg.

In this paper we follow nature's lead by concentrating on legs with elastic elements that deform during each stride. We have used such legs in machines that balance actively as they run. The functions of the elastic elements are to conserve energy associated with the bouncing motion, to reduce impact forces, and to simplify control.

Several possible leg configurations are described in figure 7–1. The Cartesian configuration using two linear joints is kinematically simple, but mechanically cumbersome. Most of the legs we have built have been of the polar configuration, having one rotary and one linear joint. Such legs are kinematically simple, and mechanically more elegant than the Cartesian arrangement. We are beginning to investigate articulated legs that use rotary joints. Articulated legs offer mechanical advantages, such as lower moment of inertia, less unsprung mass, larger range of motion, better ruggedness, and ease of construction. However, articulated legs also have added kinematic complexity and coupling between degrees of freedom. This coupling is evident from the fact that displacements of the two joints do not in general cause orthogonal displacements of the toe or hip.

Before describing the specific designs we have studied, we turn to a brief discussion of energy storage in elastic materials. For discussions of other important issues in leg design see (Hirose and Umetani 1980; Vohnout et al. 1983; Waldron and Kinzel 1983).

7.3 Mechanisms for Elastic Storage

Figure 7–2 shows the ratio of storable elastic energy to the mass of the material for several spring materials. Each material has properties that recommend it for use in leg springs, but each has drawbacks as well. Steel is an isotropic material that can easily be formed into shapes such as coils. However, steel springs have a relatively poor energy to mass ratio, about 140 J/kg. Fiberglass has about six times the energy capacity of steel, but because its fiber orientation is crucial, it is not so easily fashioned into a spring. Fiberglass is most readily used in bending as a leafspring or other beam shape.

Rubber and animal tendon have substantially higher energy capacities than steel, about 5000 J/kg. The value for animal tendon is based on Alexander's work with dogs (Alexander, 1974). Because these materials can undergo large elastic strains they can provide usable

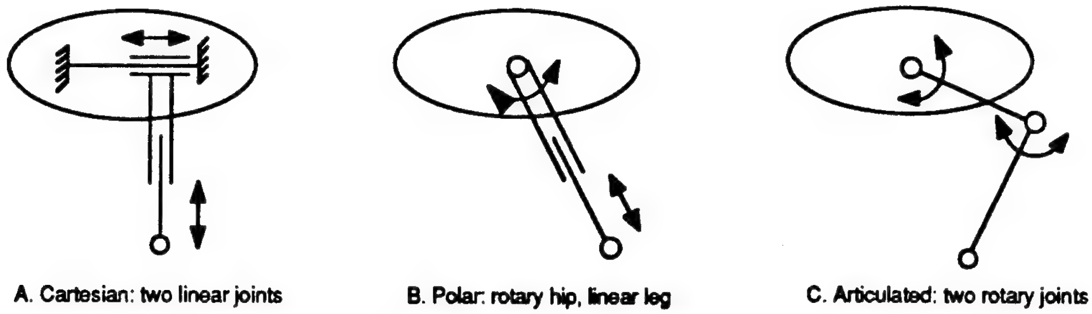

Figure 7–1: Three planar leg configurations: A) Cartesian: two orthogonal, linear joints produce independent motion. The upper joint may be troublesome because it must tolerate substantial torques while providing smooth sliding motion, and the entire mass of the leg must be accelerated in the fore/aft direction to obtain horizontal foot motion. B) Polar: rotary hip and telescoping leg provide motion in polar coordinates. The inertial loads associated with the fore/aft motion of the foot are smaller than for a Cartesian leg, because only the end of the leg moves at full foot speed. Cartesian and polar designs both require linear sliding joints, which are difficult to build with precision and resistance to side load. C) Articulated: two rotary joints avoid the friction, wear, and size disadvantages of linear joints. The drawback is the kinematic coupling of the two joints—a purely vertical or radial motion of the foot requires movement of both joints. This coupling necessitates larger ranges of joint travel than the polar leg to achieve the same foot motion, and it is difficult to resolve the springiness into the vertical direction without degrading the speed and precision of control in the horizontal direction.

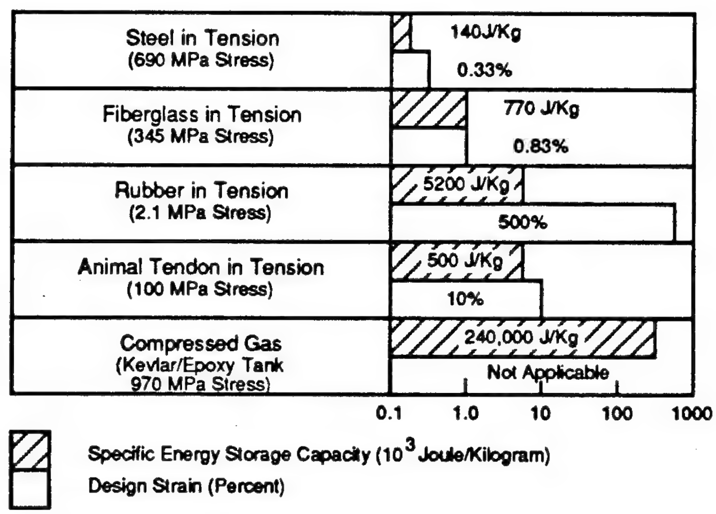

Figure 7–2: Strain energy per unit mass for various materials. The higher the value, the less mass of material needed for a given energy storage function. The strain, or relative elongation of the material, affects the design of the spring.

deflections in pure tension. The 10% strain of the Achilles tendon of an animal is compatible with the short lever arm to which the tendon is attached behind the ankle joint. For example, Alexander's data for the dog indicate that its Achilles tendon is linked to the foot so that it undergoes about one-fourth the deflection of the toe. No material usable in

human-made machines has been found with equivalent elastic properties. Rubber, because it strains about 50 times as much as animal tendon, cannot be used directly in a leg design like the dog's. Rubber is often used in the form of torsion springs and bushings that shear tangentially when loaded. Such torsion springs might be usable in rotary leg joints.

Gas compressed in a container with high specific strength has a very high energy capacity, about 240,000 J/kg. A usable gas spring requires a cylinder and piston or comparable hardware, however, which will likely weigh many times as much as an ideal container. Frictional and thermodynamic losses can be substantial. Still, gas springs may be used effectively if they are compatible with the overall design.

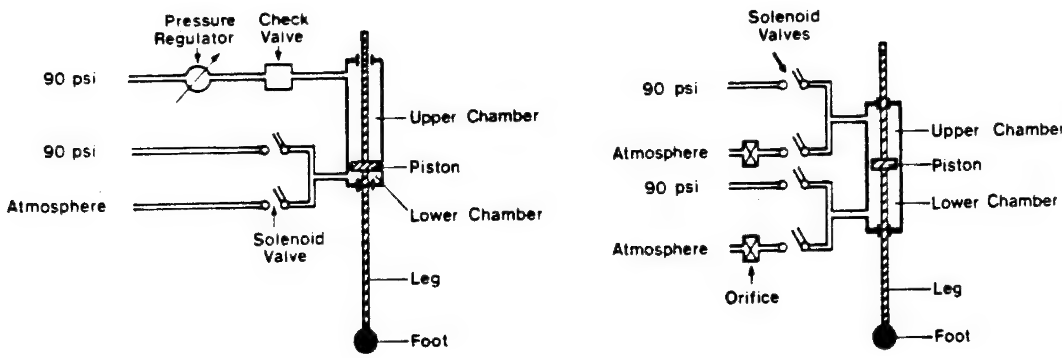

Figure 7–3: Pneumatic telescoping leg used in one-legged hopping machines. It consists of an air cylinder with a cushioned foot at one end of the rod. Electric solenoid valves control air flow to both chambers of the cylinder to extend and retract the leg, while trapped air makes the leg springy. Two pneumatic circuits were used. Left) *Top Control*—Air enters the top chamber to provide downward thrust on the foot during the support portion of the running cycle. The thrust is controlled by adjusting the pressure of the air in the top of the cylinder when the foot touches the ground. This circuit was used in a planar one-legged hopping machine (Raibert and Brown 1984). Right) *Bottom Control*—The control system adjusts thrust by regulating the pressure in the bottom chamber of the leg cylinder while a pressure regulator and check valve maintain a fixed charge of air in the top of the cylinder leg. The top chamber acts as a passive spring. To provide thrust, the control system exhausts air from the bottom chamber of the leg during support. Special quick-exhaust valves are used to dump air rapidly for maximum thrust. Bottom control is more efficient than top control because a smaller volume of air is exhausted on each cycle. This circuit was used in a three-dimensional one-legged hopping machine (Raibert et al. 1984). Sensors measured the length of the leg, the air pressure in both chambers of the air cylinder, the angle of the leg with respect to the body, and contact between the foot and the ground. The unsprung mass of the leg is 0.91 kg and moment of inertia about the hip is 0.11 kg-m^2.

7.4 Telescoping Legs

After initial attempts to use steel springs in compression, we turned to compressed gas as a mechanism for elastic energy storage in legs for running machines. A standard pneumatic

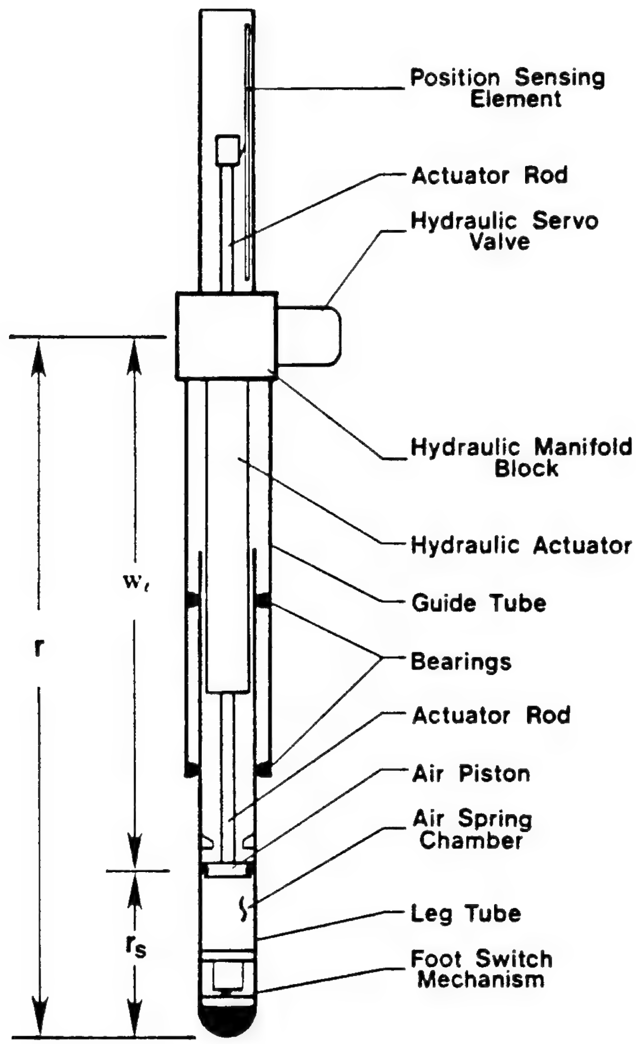

Figure 7–4: Hydraulic-pneumatic telescoping leg. See figure 7–5 for design details. A long-stroke hydraulic actuator provides controlled axial thrust and rapid retraction. An air chamber near the foot provides the spring. To reduce friction in the hydraulic actuator, all high-pressure seals are clearance seals (0.025 mm), with O-ring seals used to contain low-pressure leakage oil at the rods. Space between concentric cylinders provides paths for control and leakage flow to the lower end of the hydraulic actuator. The hydraulic actuator is servoed with a conventional high-bandwidth flow-control servo valve. The air cylinder forms the lower part of the leg and slides inside plastic guide buttons mounted in the upper leg tube. The foot includes a pneumatic check valve that allows makeup flow to the air spring, but prevents out-flow when the air spring is compressed. The hydraulic actuator has a 0.23 m travel and the air spring has a 0.10 m travel. At 17.5 MPa (2500 psi) hydraulic pressure, maximum thrust is about 950 N and maximum speed is about 2 m/s. The unsprung mass is 0.24 kg and moment of inertia of the leg about the hip is 0.13 kg-m^2. This leg design was used in a planar biped (Hodgins, Koechling, and Raibert 1986), and in a quadruped running machine (Raibert, Chepponis and Brown 1986).

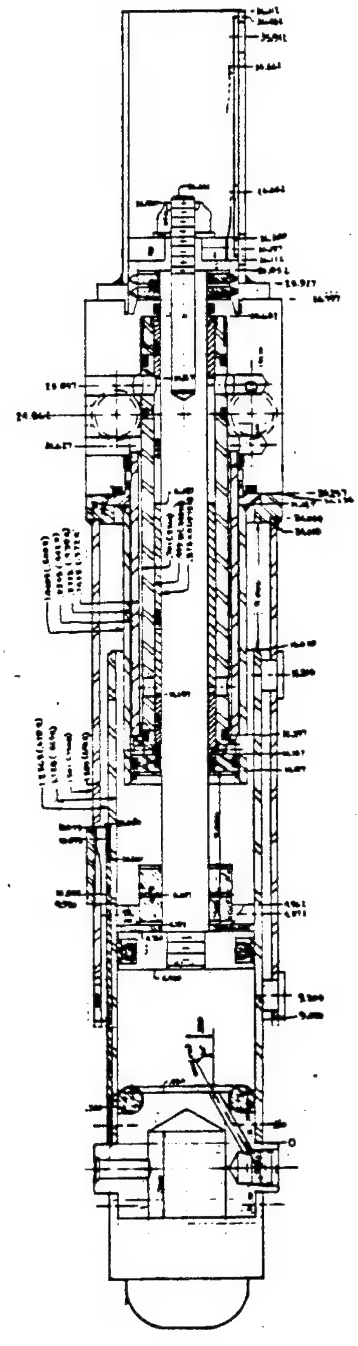

Figure 7–5: Engineering drawing of the hydraulic-pneumatic leg, showing details of the hydraulic actuator, air spring and position sensors. Leg is shown compacted in the axial direction.

cylinder with low friction seals formed the main structure of the leg in our first successful running machine. The cylinder was mounted to the body of the machine with a hinge-type hip joint, and a rubber bumper at the end of the piston rod served as a foot. A set of on-off pneumatic valves and the circuits shown in figure 7–3 provided thrust and retraction, and controlled the leg's springiness.

This pneumatic telescoping leg was used for a series of experiments on the control of machines that balanced actively as they ran (Raibert and Brown 1984; Raibert, Chepponis, and Brown 1984). One function of the spring used in these experiments was to recover part of the hopping energy during landing and return it during the subsequent upward acceleration. In an optimized design this could contribute to efficient locomotion. A second function was to provide a cushion for the upper leg and body. This cushion reduced the system's unsprung mass, the maximum loads produced by foot impacts with the ground, and the peak forces transmitted to the sprung part of the system. A third function of the spring was to simplify the control. The details of the vertical bouncing motion were determined largely by the passive oscillation of the body rebounding on the springy leg—the control system excited and modulated this oscillation but was not responsible for the details of the trajectory (Raibert 1986a).

To study running on several legs we needed a leg that could lengthen and shorten rapidly, and precisely control thrust. Rapid shortening was needed so that the recovery leg could have adequate ground clearance to swing forward while the stance leg was substantially compressed. The need to control thrust arises when coordinating the relative thrust delivered by a pair of legs that both provide support at the same time.

To satisfy these requirements—rapid retraction during recovery and precise control of thrust—we designed the leg shown in figures 7–4 and 7–5. It has a long-stroke hydraulic actuator that operates in series with a passive air spring. This leg has been used in a biped that runs and hops, and in a quadruped that trots (Hodgins, Koechling, and Raibert 1986; Raibert, Chepponis, and Brown 1986). Although this design has been used successfully in experiments, it has several limitations:

- The leg is relatively heavy.
- Seal leakage and friction during compression degrade the resilience of the air spring.
- The sliding joint is mechanically complex and bulky, and subject to wear and looseness.
- Measuring leg length requires a long, specially made sensor.
- Wires to the foot must go through slack cables that are vulnerable to a variety of hazards.
- The moment of inertia of the leg is substantially larger than desired.

These limitations have motivated us to explore articulated legs that use only rotary joints.

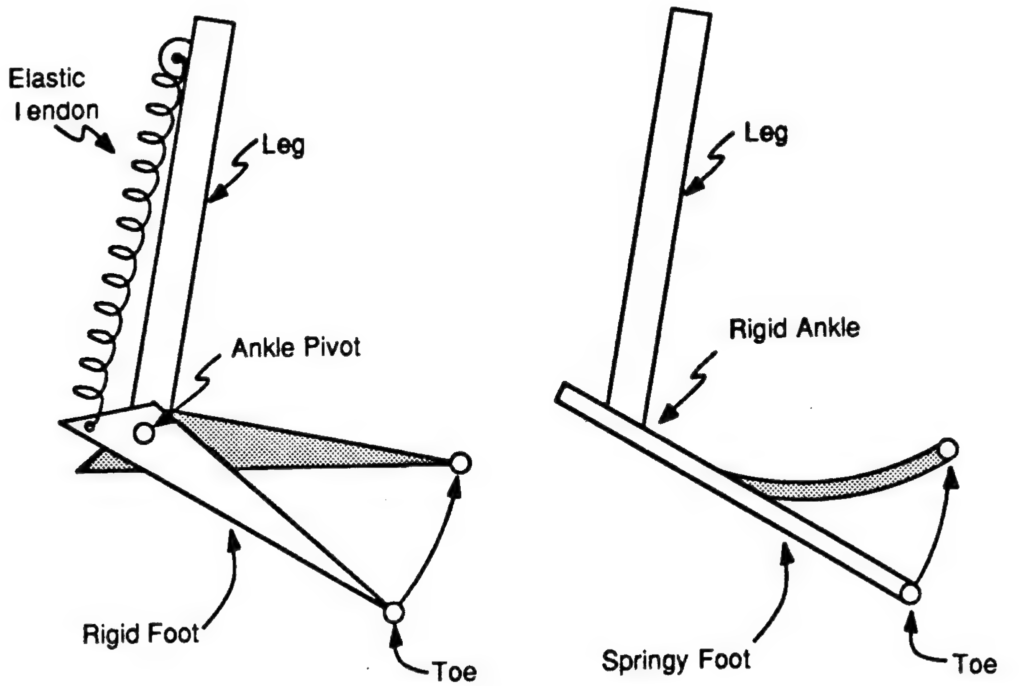

Figure 7–6: Articulated legs. Left) An anthropomorphic design uses a hinged, inflexible foot with a springy tendon. Right) Alternate design employs bending of an elastic foot to provide springiness.

7.5 Articulated Leg

A fundamental objective in the design of an articulated leg has been to provide compliance in the radial direction but not in the tangential direction. Such a leg would be functionally similar to the telescoping legs we have already built. As a first step, we considered two methods for incorporating springiness in the leg, as shown in figure 7–6. The somewhat anthropomorphic design shown on the left side of figure 7–6 incorporates a rotary ankle joint that connects a rigid foot to the leg. An elastic tendon acts through a lever behind the ankle to provide the needed downward force and compliance at the toe. A suitable tendon material for such a design has not been found.

The design shown on the right side of figure 7–6 employs a noncompliant tendon that acts on a leafspring foot. Energy can be stored in the bending of the foot. The design combines the structural and elastic functions into a single unit, minimizing mass. Because of the distributed nature of a leafspring, the effective unsprung mass is small. This results in low impact forces during running, minimizing energy losses.

To test the springy-foot concept we built the one-legged machine, or "monopod" shown in figure 7–7. This machine is constrained to operate in the plane by a tether mechanism that permits forward and vertical translation and pitch rotation. The foot is actuated about a rotary ankle joint by a linear hydraulic actuator that pulls on the foot through an inelastic tendon. This actuator is located at the hip to minimize the rotational inertia of the leg. A second hydraulic actuator drives the swing motion of the hip. The machine is designed

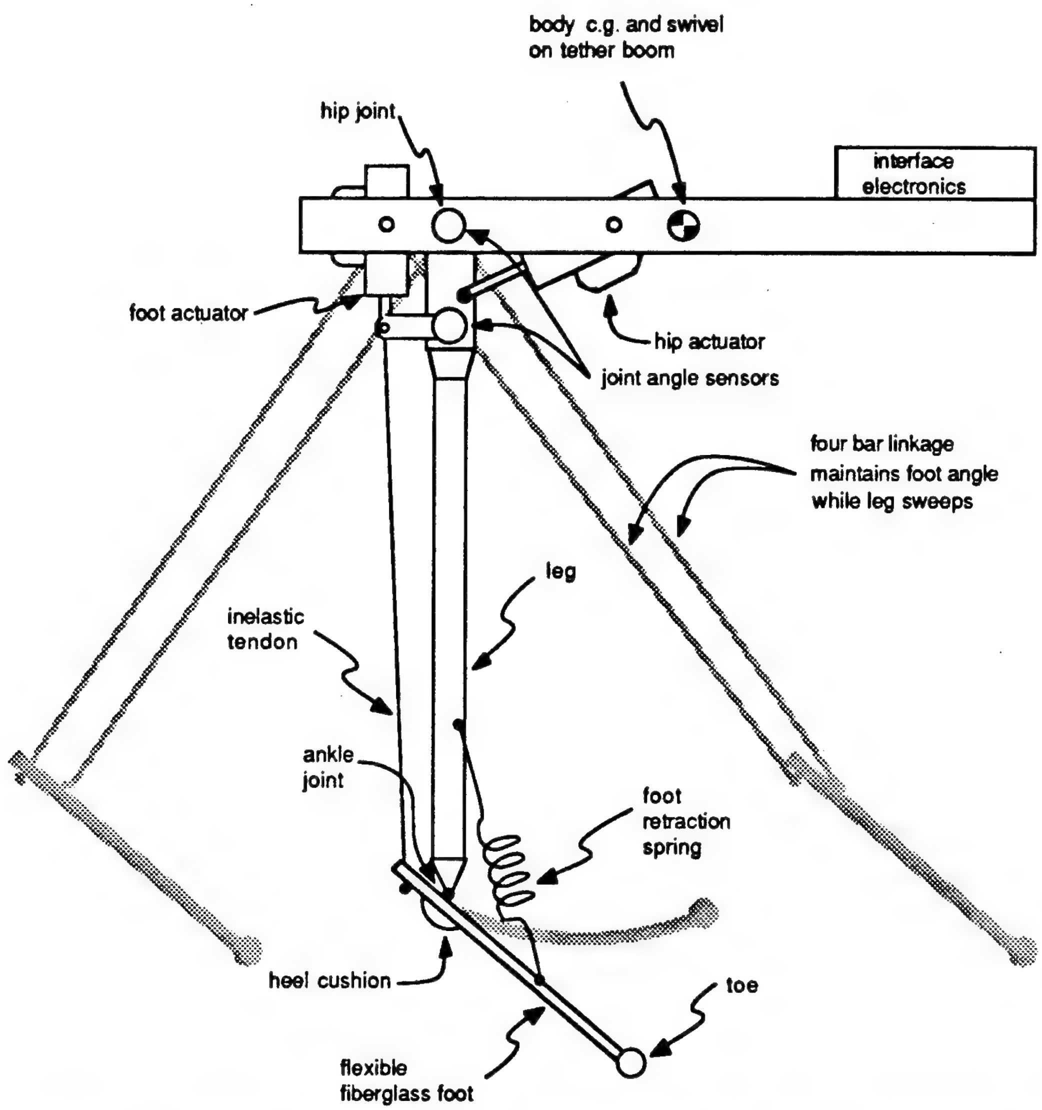

Figure 7–7: Diagram of monopod with articulated leg. The foot is a leafspring that deflects during hopping. The ankle is actuated through an inelastic tendon and hydraulic actuator mounted at the hip. A retraction spring attached to the foot maintains tension in the tendon. The linkage makes the foot angle with respect to the body nearly independent of the hip angle. Potentiometers measure the two joint positions and foot deflection. The leg is intended for planar operation. The unsprung mass is 0.063 kg and moment of inertia of the leg about the hip is 0.097 kg-m^2.

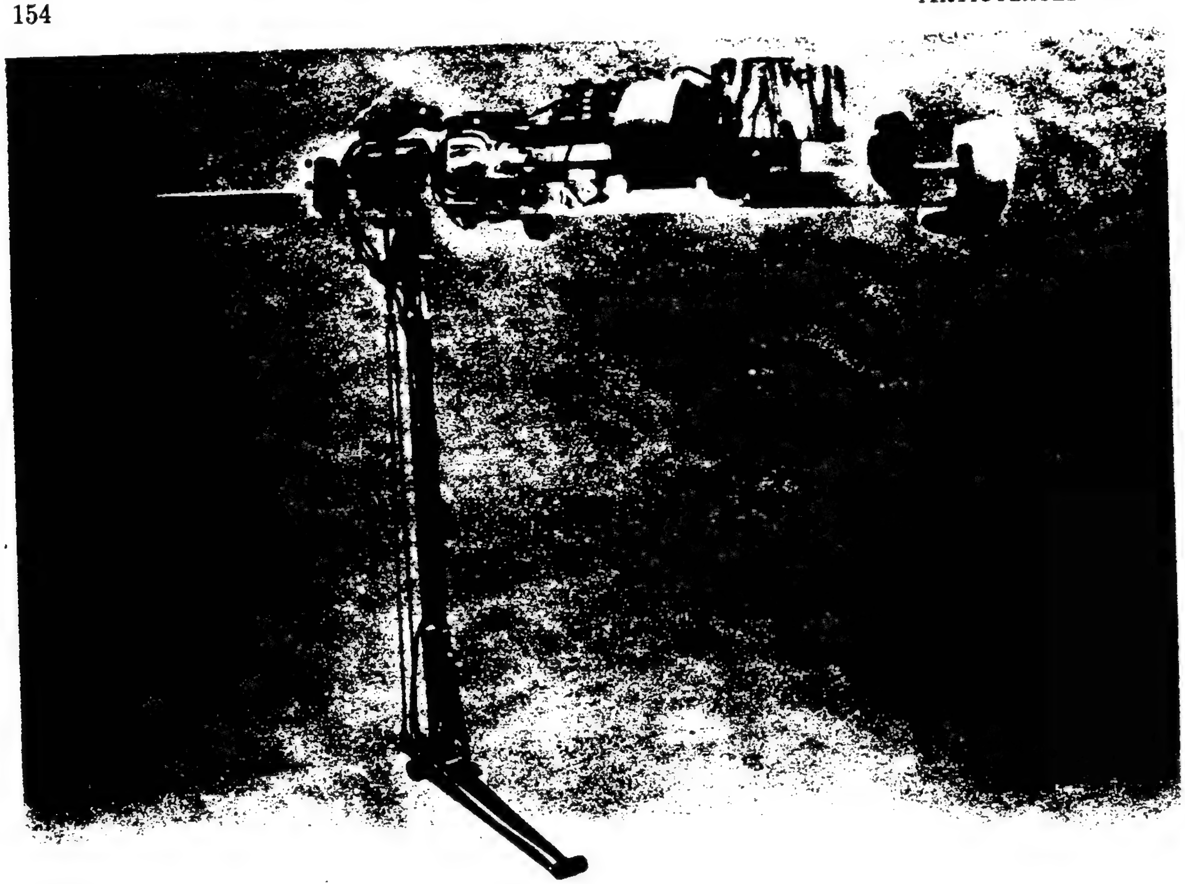

Figure 7–8: Photograph of the monopod. A long beam attached to the body allows mounting of weights to adjust location of the center of gravity and body moment of inertia. Metal tubing on frame carries hydraulic fluid which is fed through swivels to actuators. The aluminum arm and potentiometer on the foot are mounted to measure foot deflection.

to run on its toe, which is located below the center of mass of the machine when the leg is vertical. The hip is offset from the center of mass by a distance roughly equal to the offset of the ankle with respect to the toe, so the leg is nominally vertical when the hip is centered. The four-bar linkage formed by the leg, heel lever, body, and tendon keeps the orientation of the foot with respect to the body nearly constant as the leg swings fore and aft. Appendix A gives the physical parameters for the machine. Appendix B describes in detail the kinematics of the machine.

The number of sensors used for control of the monopod is small. Rotary potentiometers measure the angles of the hip and ankle joints. A rotary potentiometer connected to the ankle and toe measures deflections of the foot and contact with the ground. Rotary potentiometers at the base of the tether boom measure the horizontal and vertical positions of the center of the machine.

The control system for the monopod uses the same three-part control decomposition that we have used many times in the past. Hopping height is controlled by shortening the tendon actuator during stance. This excites the spring-mass system formed by the springy

foot and the body. The body's pitch attitude is controlled by applying hip torque during stance in proportion to body pitch angle and pitch rate errors. Forward speed is controlled by setting the position of the toe with respect to the body at touchdown. A number of features described later in this chapter have been incorporated to improve performance.

7.6 Monopod Experiments

The monopod is being used to evaluate the various features of the mechanical design and to explore what impact an articulated leg has on the control of dynamic legged locomotion. In each experiment an operator starts the machine by aligning the body and leg, and then dropping the system from a few inches to start it bouncing. Then the operator manipulates a joystick to specify desired running speed. During these experiments the operator may adjust a number of parameters to examine various aspects of the mechanical system or the control.

Figure 7–9 shows data from one of the best runs the machine has made, with a maximum running speed of 2.3 m/s (5.1 mph) averaged over 16 m. The machine tracked the desired speed with an error of about 0.2 m/s at steady speed. The body's pitch angle error was kept below 5 deg, typically with a nose-down posture.

Early experiments were performed using 1500 psi hydraulic supply pressure, a relatively low value, to minimize the potential for damage to the machine. We found that this pressure was inadequate for the tendon actuator to maintain the foot's position under maximum loading during stance. This permitted the heel of the foot to collide with the ground. The 1500 psi of hydraulic pressure can produce only about 1100 N of tension in the tendon, while peak ground reaction forces are around 1600 N. Figure 7–10 shows this condition. The heel impact is shown by the disturbance in the foot deflection curve just after its peak. Comparison of the foot angle and its setpoint indicates that the actuator is being backdriven by the ground contact force. Impact of the heel causes a substantial disturbance in pitch, shown particularly in the pitch velocity signal, and results in severely nose-down running. Figure 7–11 shows data for a run with the hydraulic supply pressure set to 3000 psi. Whereas the heel no longer strikes the ground at 3000 psi, there are severe oscillations in thrust and body pitch, as explained below.

The control system delivers thrust during stance by moving the setpoint for the angle of the lever at the rear of the foot (θ_{heel}). This lengthens the zero point of the foot spring, adding energy to the system. The foot actuator is controlled during stance with a linear servo of the form

$$t = -k_p(\theta_{\text{heel}} - \theta_{\text{heelsp}}) - k_v\dot{\theta}_{\text{heel}} \tag{7.1}$$

where

t	is the signal to the hydraulic servo valve,
θ_{heelsp}	is the setpoint for the heel angle, one of two preselected positions,
$\dot{\theta}_{\text{heel}}$	is the time derivative of θ_{heel}, and

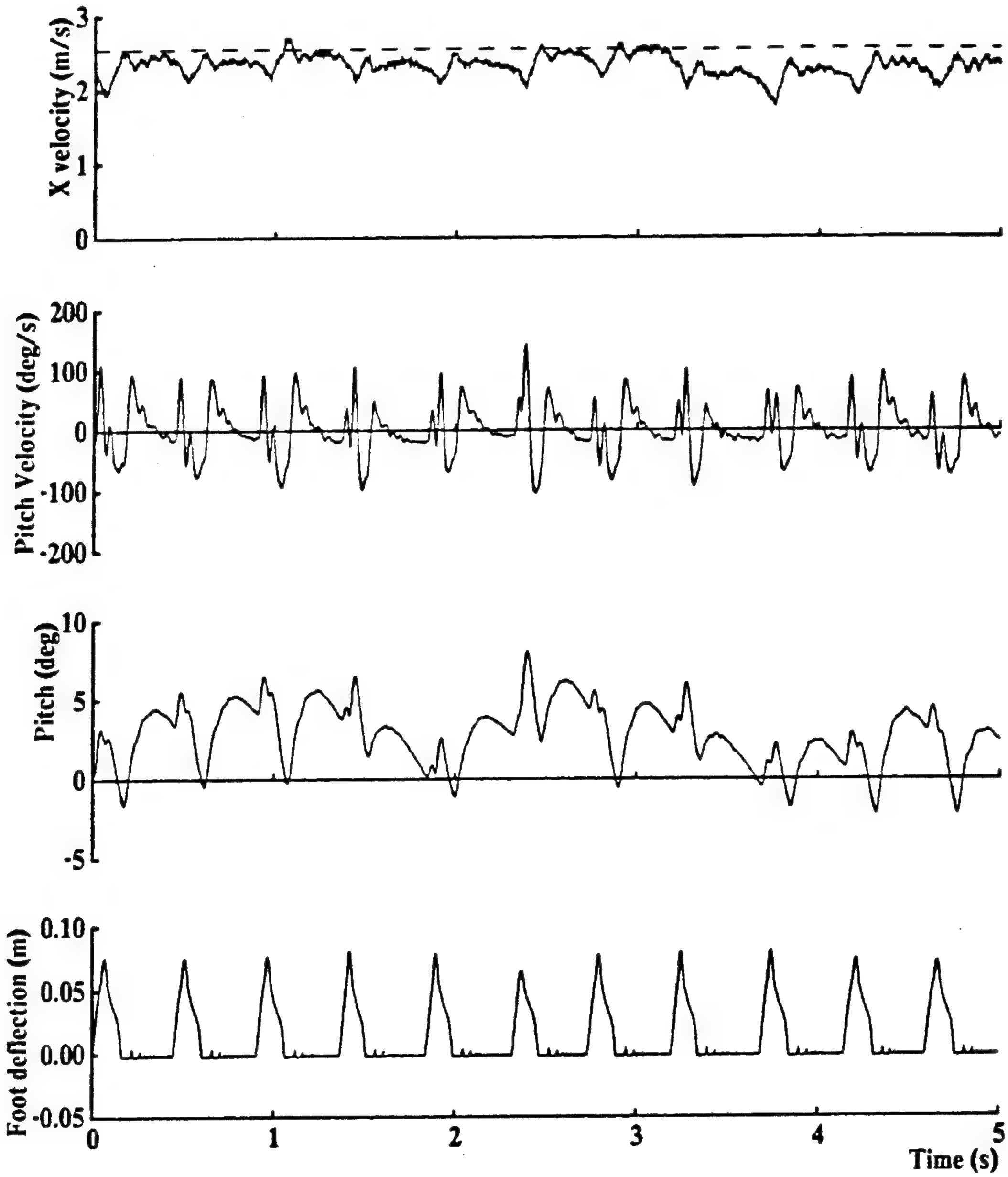

Figure 7–9: Running at constant speed, 11 cycles. Periods of stance can be distinguished from the lower graph of foot deflection. The single peaks in foot deflection indicate the absence of heel impacts with ground. Positive spikes in pitch velocity show forward pitching when toe strikes ground. Top graph shows that forward running speed $\dot{x}$ is near the desired value, which is shown by the broken line. (M.155.1)

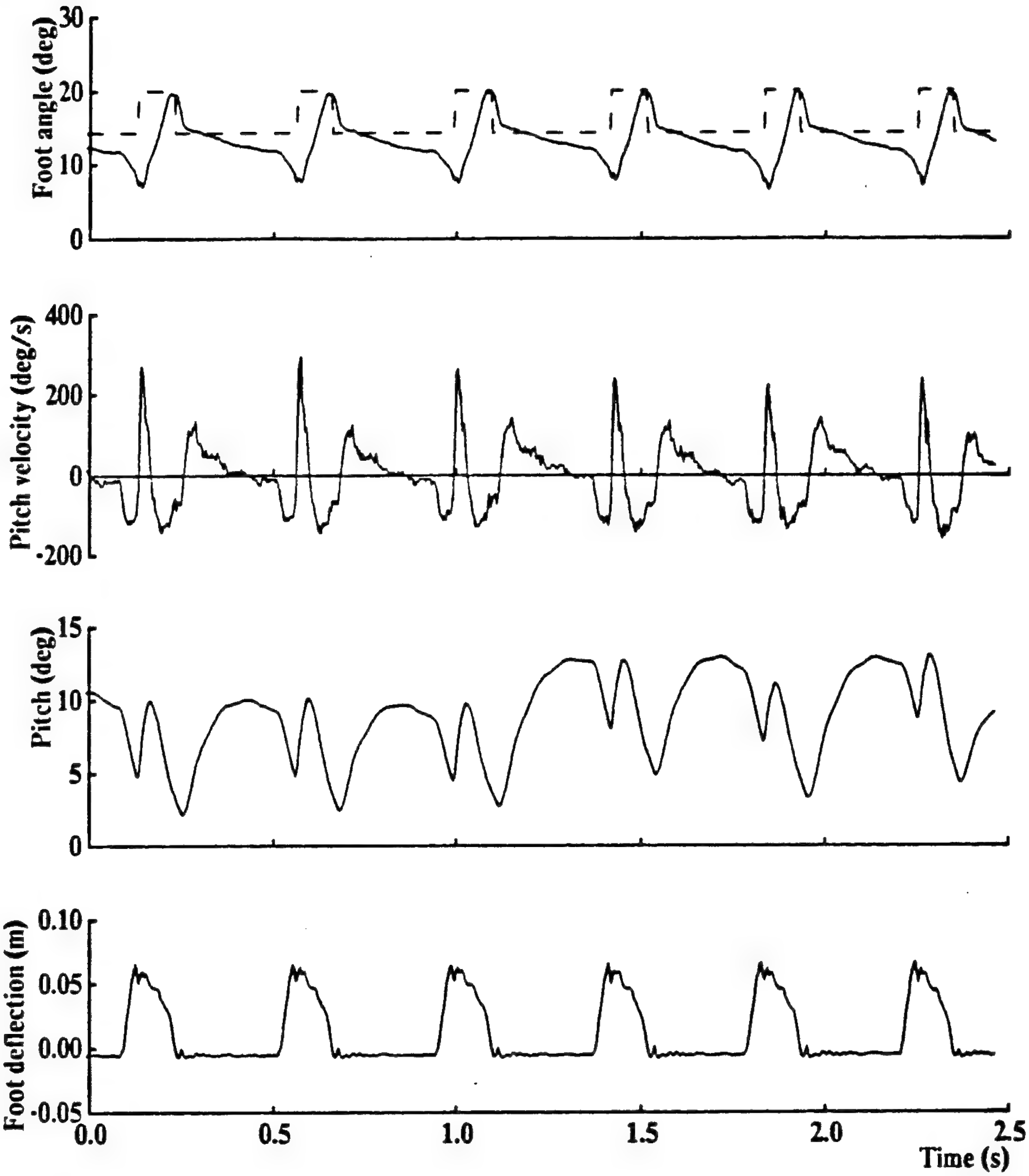

Figure 7–10: Running data for the monopod with the heel hitting the floor, 6 cycles. The disturbance in foot deflection just after the peak indicates impacting of the heel on the ground. The positive spikes in pitch velocity correspond to striking of the heel on the ground, which causes a substantial forward pitching moment. Graph of foot angle and setpoint (broken line) shows that the ground force is driving the foot away from the setpoint when foot deflection is large, due to inadequate actuator hydraulic pressure. Upward step in setpoint is where thrust begins. (M.190.2)

k_p and k_v are position and velocity gains.

The data shown in figure 7–9 were produced using this servo.

At higher hydraulic supply pressures, the thrust servo oscillates as shown in figure 7–11. The oscillation results apparently from the relative loss of mechanical damping at the higher pressures and loads. We have been unable to solve the problem by simply reducing position gain or increasing the gain on the velocity signal, which is obtained by differentiation. One way to eliminate the oscillation is to control thrust with a timed signal of preselected magnitude, rather that with a position servo. In this scheme we manually control the timing and magnitude of the signal sent to the foot actuator each stance period. This method has been employed successfully.

Increased hydraulic pressure has produced oscillations in the attitude control servo as well, as shown particularly by the pitch velocity graph in figure 7–11. These oscillations cause the machine to leave the ground with substantial, erratic pitch velocity. Because pitch velocity remains constant during flight, large errors in pitch position can accumulate before the next stance period. Further work is needed to determine how to damp these oscillations. The use of pitch acceleration feedback is one possibility being explored.

Several features have been added to the monopod control system to improve attitude control in general. To minimize the forward pitching that naturally occurs when the foot strikes the ground with the machine moving forward, the control system generates a signal to start sweeping the leg to match the relative ground speed just before the foot strikes the ground. This is done about 20 ms before contact. Time before touchdown is computed on the basis of vertical velocity, vertical height and the vertical component of leg length. This anticipatory leg sweep term reduces the attitude disturbance that occurs at touchdown. This same sweep term is incorporated into the pitch-control servo

$$t = -k_p\phi - k_v\dot{\phi} - k_{\mathrm{sweep}}\dot{\theta}_{2\_nom} \tag{7.2}$$

where

t	is the signal to the hip actuator valve,
ϕ	is pitch angle,
$\dot{\phi}$	is pitch velocity,
$\dot{\theta}_{2\_nom}$	is the nominal leg sweep velocity based on the machine kinematics and forward speed at touchdown, and
$k_p, k_v, k_{\mathrm{sweep}}$	are gains.

The sweep gain, k_{sweep}, is determined by comparing the unloaded, steady-state, leg-sweep velocity with the corresponding signal to the hip servo valve. This additional term allows the leg's sweeping motion to proceed without errors in the body's pitch angle and pitch rate, and should reduce steady-state errors, or permit the use of lower attitude-control gains.

One advantage of the leafspring foot is that its unsprung mass is very low compared to that of the telescoping legs on previous machines. Unsprung mass is the mass whose kinetic energy is lost at touchdown. The bouncing efficiency of a machine, that is the fraction

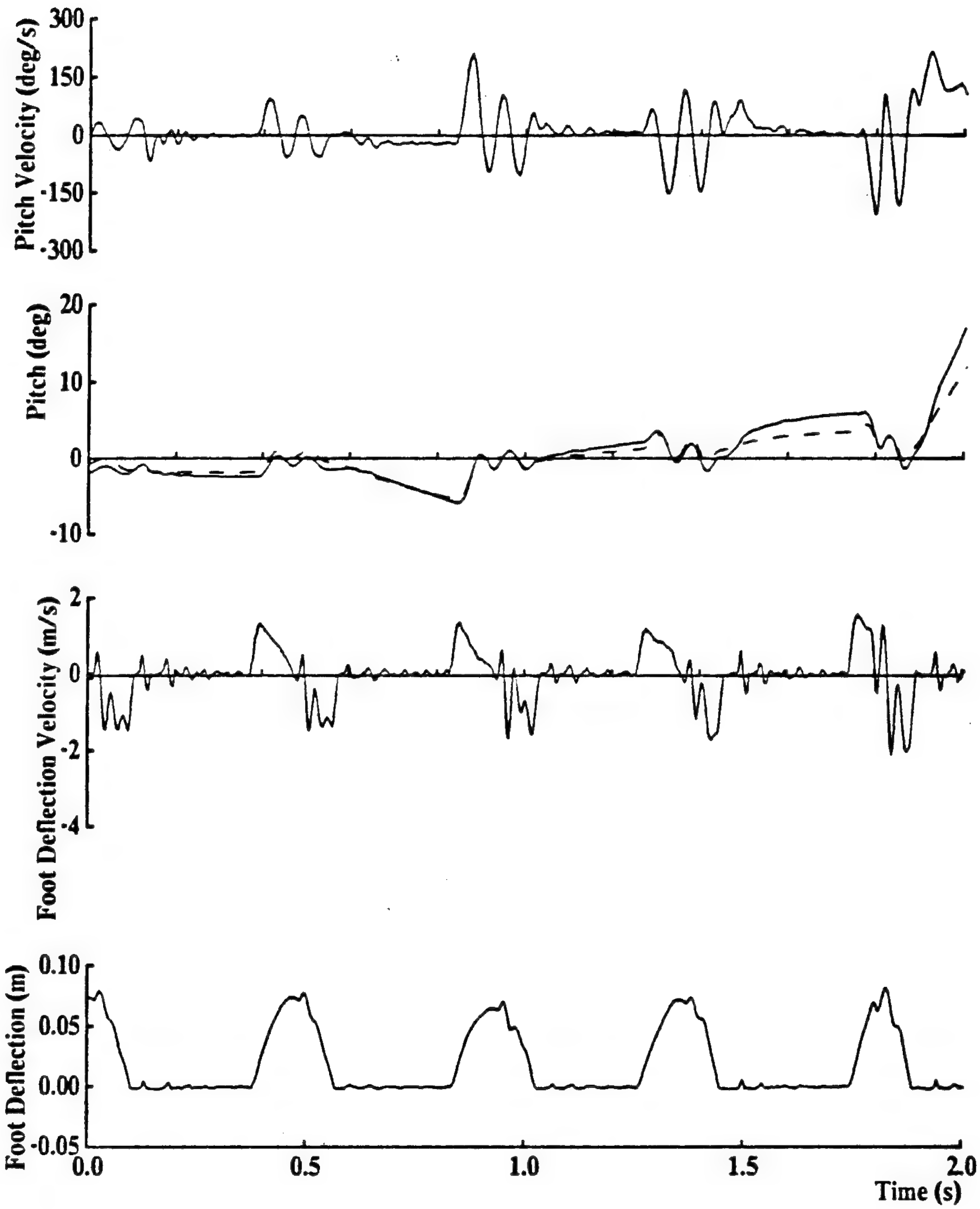

Figure 7–11: Oscillations in foot deflection and pitch. Hydraulic supply pressure of 3000 psi results in oscillations in foot deflection and pitch accentuated in the two velocity graphs. The pitch graph shows body pitch angle compared to the weighted-average system (body plus leg) pitch angle, shown by the dashed line. The average value goes through smaller pitch excursions than the body, as discussed in the text. (M.225.2)

of the circulating energy recovered from one hopping cycle to the next, is limited by the unsprung mass, as given by

$$\eta_{max} = (1 - \frac{M_{un}}{M_{sys}})^2 \tag{7.3}$$

where

η_{max}	is the maximum theoretical bouncing efficiency,
M_{sys}	is the mass of the whole system, and
M_{un}	is the unsprung mass.

This equation accounts for the losses in kinetic energy of the system that occur at touch-down and lift-off. It ignores, of course, the energy needed for swinging the leg and for the various control functions, and frictional losses. Based on this equation and an unsprung-mass ratio of 0.008 for the monopod, we would expect a negligible loss (1.6%) in bouncing efficiency due to foot impacts. We tested the efficiency of the monopod's springy foot by measuring the behavior as the system bounced passively after being dropped from a height of several inches. The bouncing height and spring energy on successive bounces indicate an efficiency of about 0.67.

7.6.1 Mechanical Weaknesses

Several mechanical weaknesses have been found in the monopod design. Failure of the tendon, the "Achilles" tendon, has occurred on numerous occasions, usually at terminations or bends in the cable. The original 1200 N (1.2 mm) aircraft cable (two strands in parallel) was replaced first by 1500 N cable, then by 2200 N cable, and finally by 1.4 mm dia. music wire having a breaking strength of around 3500 N, giving 7000 N strength for the two strands. Peak tension in the tendon, based on measured foot deflections, is around 1600 N for typical running. The fiberglass foot has broken and been replaced twice. Failures have resulted from shearing between the fiberglass laminations at the heel. The most recent prototype has been strengthened in this area. Strain gauges to measure foot deflection have failed repeatedly, due to delamination or breakage of the foil traces. Although the peak strains are high, about 1.2%, they are within specifications for the gauges and adhesives. A rotary potentiometer on the foot now provides deflection information. The leg tube has also failed twice, once near the hip, once near the ankle hinge. Both areas have been strengthened. We expect to expose other weaknesses as we push the machine toward better performance.

7.6.2 Discussion

A difficulty with the articulated legs is that deflection of the foot causes motion of the toe that is not along the axis of the leg. Assuming the toe is rigidly fixed to the ground during stance, the ankle will move along a path that is approximately circular, centered at the toe (figure 7–12, left). During vertical bouncing this causes the ankle and the lower

part of the leg to have a horizontal component of motion. This kinematically induced horizontal motion increases with the foot's angle α with respect to the ground, and adds to the effective unsprung mass of the foot. It also requires an adjustment at the hip to avoid disturbing of the body attitude. A longer foot reduces α for the same vertical deflection, but at the expense of additional foot mass. Thus there is a tradeoff between a long foot that minimizes α and a short foot that minimizes mass. In principle one could compensate for this horizontal deflection by introducing an additional pair of leafsprings (figure 7–12, right). Properly designed, such a mechanism could deflect with a nearly vertical motion of the toe, although it might be difficult to build.

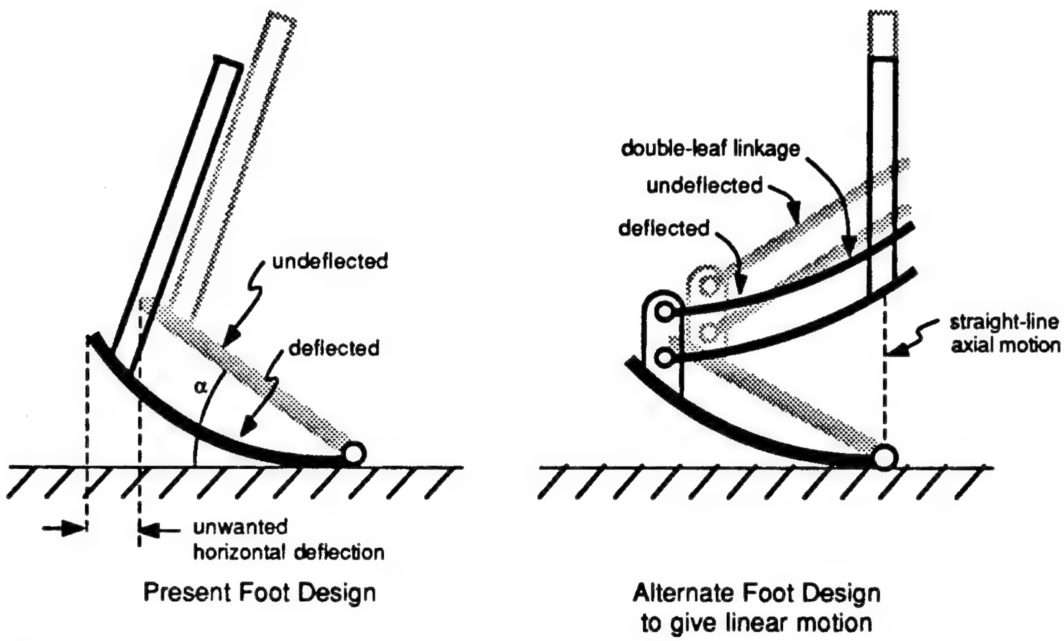

Figure 7–12: Left) Deflection of leafspring foot introduces an undesirable horizontal motion of the toe with respect to the ankle. Right) Introduction of an additional pair of leafsprings compensates for the horizontal motion, yielding a nearly vertical motion of the toe.

Because the tangential and radial motions of the toe depend on the positions of both the hip and ankle joints, there is heavy coupling between the sweeping motion of the hip and the thrusting motion of the ankle, particularly when the foot is steeply angled with respect to the leg. There are several possibilities for dealing with this interaction. One is to calculate a nominal correction for a standard bounce, and to adjust the hip's motion during stance accordingly. A second approach is to use a force servo on the pitch attitude control. The servo could use differential hydraulic pressure or an explicit measure of the hip actuator force output. Another approach would be to feed back the angular acceleration of the body for control. A fluid-inertia angular accelerometer, that will be rugged and insensitive to cross-axis effects, is being developed for this purpose. We have not found suitable commercially available angular accelerometers.

The ratio of leg moment of inertia to body moment of inertia is a significant parameter

in running. During ideal, one-legged running, the body and leg counteroscillate so that the total angular momentum of the system remains zero. On previous one-legged machines, the leg moments of inertia were relatively small, so the angular momentum of the legs could be ignored in the attitude-control algorithm with little adverse affect. During two-legged running, the legs oscillate 180 deg out of phase, so the net angular momentum is approximately zero and may be neglected. On the monopod, the leg angular momentum is significant. Therefore we have recently incorporated an attitude-control algorithm that tries to servo to zero the total angular momentum and the weighted-average angular position of the body and leg. We define an average pitch angle

$$\bar{\phi} = \left(1 - \frac{J_{\text{leg}}}{J_{\text{sys}}}\right)\phi + \frac{J_{\text{leg}}}{J_{\text{sys}}}\theta_{\text{leg}} \tag{7.4}$$

where

ϕ	is the body pitch angle wrt horizontal,
θ_{leg}	is the leg angle wrt vertical,
$\bar{\phi}$	is the weighted-average pitch angle,
J_{leg}	is the moment of inertia of the leg about the center of gravity of the system, and
J_{sys}	is the moment of inertia of the system about its center of gravity.

We then use $\bar{\phi}$ and its time derivative in place of pitch as a basis for a linear servo. This should minimize the attitude-control "effort", and result in a smooth, natural motion. Figure 7–11 shows that the weighted-average angular position goes through smaller oscillations than the body pitch angle, as expected due to counter rotation of the body and leg. Further tuning is needed to verify the success of this approach.

Unlike previous machines, the monopod is not symmetrical in the fore/aft direction. It has demonstrated a clear preference for traveling in the "forward" direction, that is, the direction in which the toe points. Because of the asymmetry, the mass of the leg is not aligned with the center of mass of the machine as it is with the telescoping legs used on the other machines. To offset the mass of the leg, we have a compensating mass on the body, so the center of mass of the entire machine is at the nominal center, the point above the toe.

The difference in leg-spring design between the monopod and previous machines raises the issue of how spring characteristics affect machine performance, and what characteristics are desirable. Some desirable characteristics are obvious: springs should be light in weight, compact, and rugged. They should have maximum energy storage capacity, good resilience, mechanical simplicity, and be easy to build. Other characteristics, however, are not obvious. Is a spring with a linear force/deflection characteristic preferable to the nonlinear characteristic of a gas spring? Should the spring be preloaded, and if so, how much? The system that is simplest to analyze is a linear spring with a preload that just equals the weight of the machine. In this case, the preload effectively negates the effect of gravity, and the system undergoes one-half cycle of pure harmonic motion during stance, assuming the spring acts vertically. Stance time will not vary with hopping height in this case. Whereas the nonlinear force/deflection function of a gas spring makes it analytically difficult, its

inherent *hardness*, due to the asymptotically infinite forces near the end of travel, provides resistance to bottoming—a distinct advantage.

7.7 Alternative Design for Articulated Leg

As a possible solution to the problem of the strong coupling between the sweeping motion of the hip and the thrusting motion of the ankle, an alternative foot design is proposed as shown in figure 7–13. This design places the toe on a platform that elevates it with respect to the ground. The platform is like a hoof. As mentioned previously, one solution to the coupling problem is to minimize the foot angle α. The proposed design reduces the foot angle without causing the ankle to collide with the ground. Although the motion of the ankle about the toe is still along a circular arc, it is now symmetrical with respect to the horizontal line passing through the toe joint. Therefore it induces less horizontal motion of the ankle.

The basic idea of this design was inspired by the mechanism of a horse's hoof (figure 7–14). Impact of the foot against the ground bends the fetlock joint and stretches an elastic ligament. The fetlock snaps back when the foot leaves the ground. Such motion induces an upward push to the leg. Because a suitable artificial tendon-like material for such a design has not been found, the fiberglass leafspring will be used in the new design as the energy storage element.

We hope to replace telescoping legs with articulated legs in future running machines. However, because of its inability to retract substantially, the current articulated leg design is not adequate for running machines with more than one leg. With a single leg, the swing phase occurs during flight when the foot is clear off the ground. But for a machine with more than one leg, the idle leg must remain clear of the ground while the other leg is in stance—the idle leg must be able to shorten to less than the shortest length seen by the support legs during stance.

One way to obtain large retraction is to add a knee-like joint. The ankle pivot and tendon could then be eliminated because vertical thrust could be obtained by a combination of knee and hip motions. However, a complication then arises due to a kinematic coupling between hip and knee joints—both purely vertical movement and purely horizontal movement require use of the same joints. Therefore, use of the ankle as the primary actuator while using the knee joint mainly for gross changes in leg length during flight seems to be the best choice. However, the final mechanical design of such a ankle-knee articulated leg has not been considered yet, and it will be the next step in the development of the project.

7.8 Planerizer

We have designed a mechanism that will permit the Monopod to run on a treadmill, while constraining its motion to the plane. The main function of the planerizer is to restrict motion of the Monopod in all degrees of freedom, except those in the sagittal plane—fore and aft, up and down, and pitch rotation. In fact, the function of the planerizer is same as of the tether boom used to constrain the planar biped described in earlier chapters of this report. However, the tether boom turns the machine as the machines moves fore and aft. If the monopod were to travel forward on the treadmill while tethered by a boom, the legs would no longer sweep in the direction of the moving belt.

The planerizer should be rigid enough to eliminate non-planar motions, and should have minimum influence on the dynamics of the Monopod. Low friction is thus very desirable. It is also important that the size of the planarizer permits it to be mounted on the treadmill. The major criteria for design of the planarizer are that it

- Allows motions within the sagittal plane, while preventing motions out of the plane.
- Does not disturb dynamics.
- Adequate size.

We have considered several designs, some of which are shown in figure 7–15—the arm structure, the X-Y table, and the linear-sliding boom. However, none of these designs satisfied all the criteria mentioned above. For example, the arm structure is vulnerable to the side thrust applied by the robot due to its long arm, and requires a bulky and heavy arm in order to be rigid enough. Furthermore, the distribution of inertia is even only over a limited range of orientations, and otherwise it disturbs the dynamics of the robot. The X-Y table satisfies the first and third criteria, but it does not satisfy the second criterion because of its uneven distribution of mass. The linear-sliding boom meets all the criteria. However, the arm has to be relatively long, and the space available is thus the limiting factor.

The schematic of the design we have selected is shown in figure 7–16. This mechanism operates in Cartesian coordinates. It consists of two vertical linear slides and rails, two horizontal linear slides and rails, pulleys, and cables. The running machine is to be mounted on the vertical linear slide #1 which rides on the vertical rail #1. The vertical rail #1 in turn is mounted on the two horizontal linear slides as shown, whereas the vertical rail #2 is stationary. The force transmitting elements are the pulleys and cables. They are mounted in such a way that the vertical movement of vertical slide #1 will cause exactly the same movement in vertical slide #2, but the horizontal movement will not affect the vertical slide #2. Therefore the inertia of the machine's vertical movement can be adjusted to be the same as the inertia of horizontal movement by adding weights on the vertical slide #2. The major possible drawback is friction in the cables and pulleys. We hope to keep friction low through the appropriate design and selection of components. The design permits measurement of location of the machine by measuring the rotation of the pulleys using rotary digital encoders. The planerizer is currently under construction.

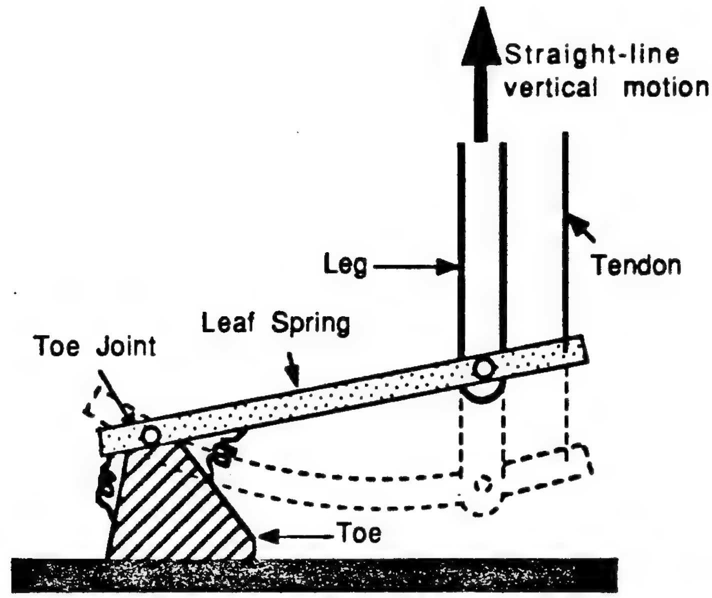

Figure 7–13: Drawing of alternative articulated leg design. This design places the toe on a hoof-like platform that elevates it with respect to the ground. It reduces the foot angle α without causing the ankle to collide with the ground, and thus minimizes the coupling between the vertical motion and the horizontal motion.

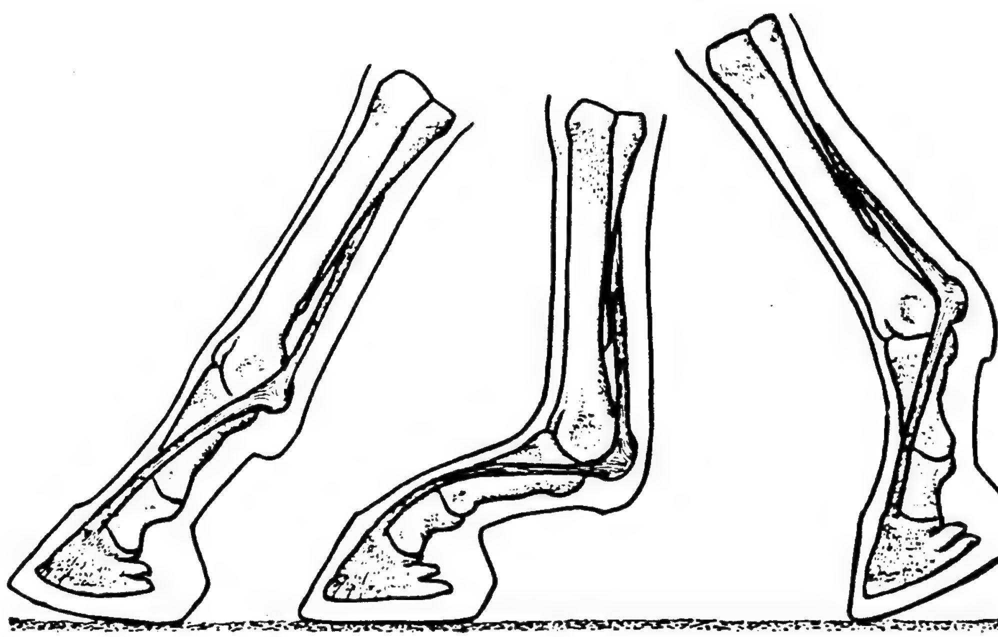

Figure 7–14: Horse's hoof. The basic idea of the alternative leg design was inspired by the mechanism of a horse's hoof. The hoof elevates the toe so the joint can move lower without touching the ground. Figure reprinted from (Hildebrand 1960).

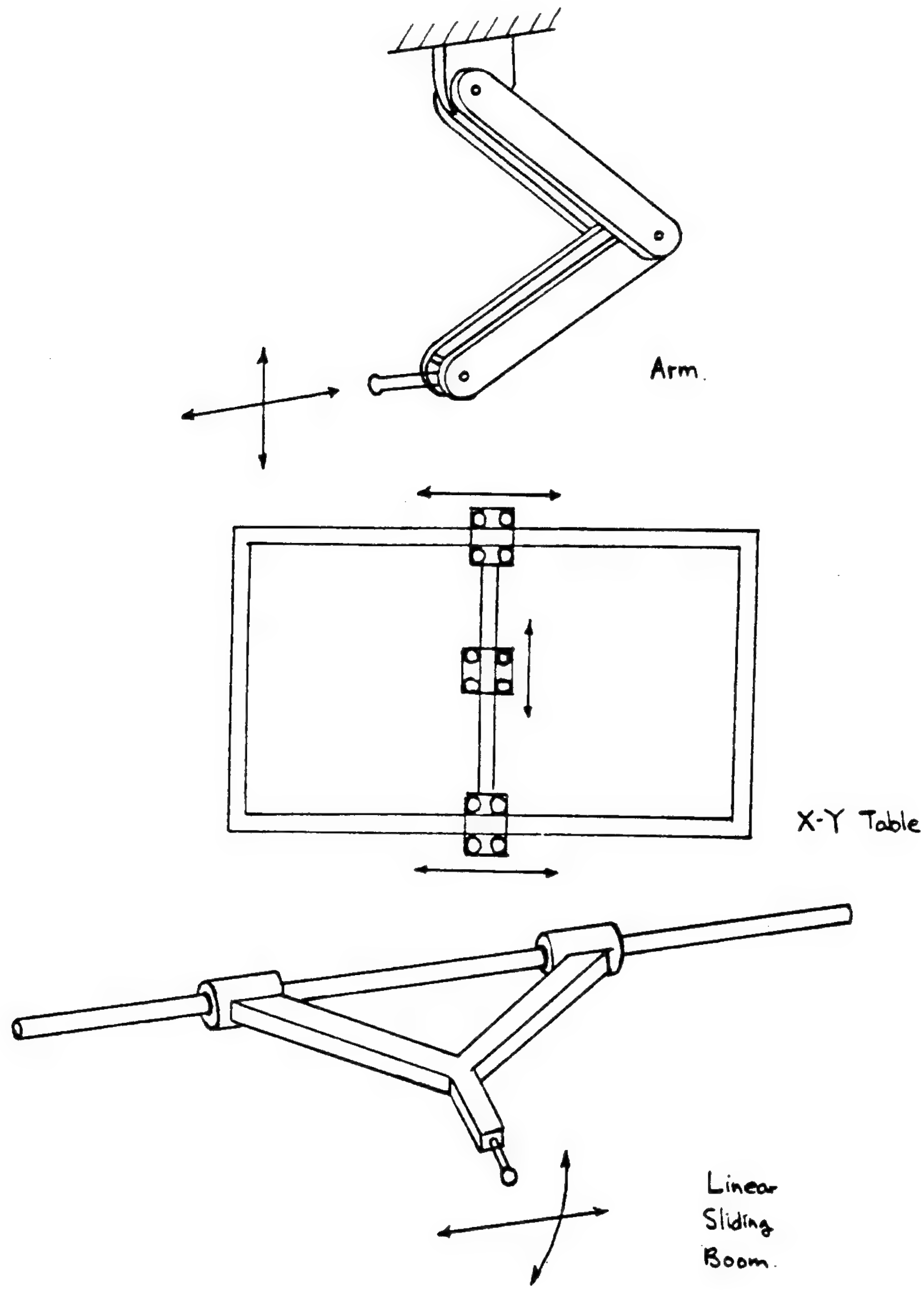

Figure 7–15: Three designs of planerizer: A) Arm structure: It is vulnerable to the side thrust applied by the robot due to its long arm, and requires a bulky and heavy arm in order to be rigid enough. The distribution of inertia is even only over a limited range of orientations, and otherwise it disturbs the dynamics of the robot. B) X-Y table: The distribution of inertia is uneven. C) Linear-sliding boom: The arm has to be relatively long, and the space available is thus the limiting factor.

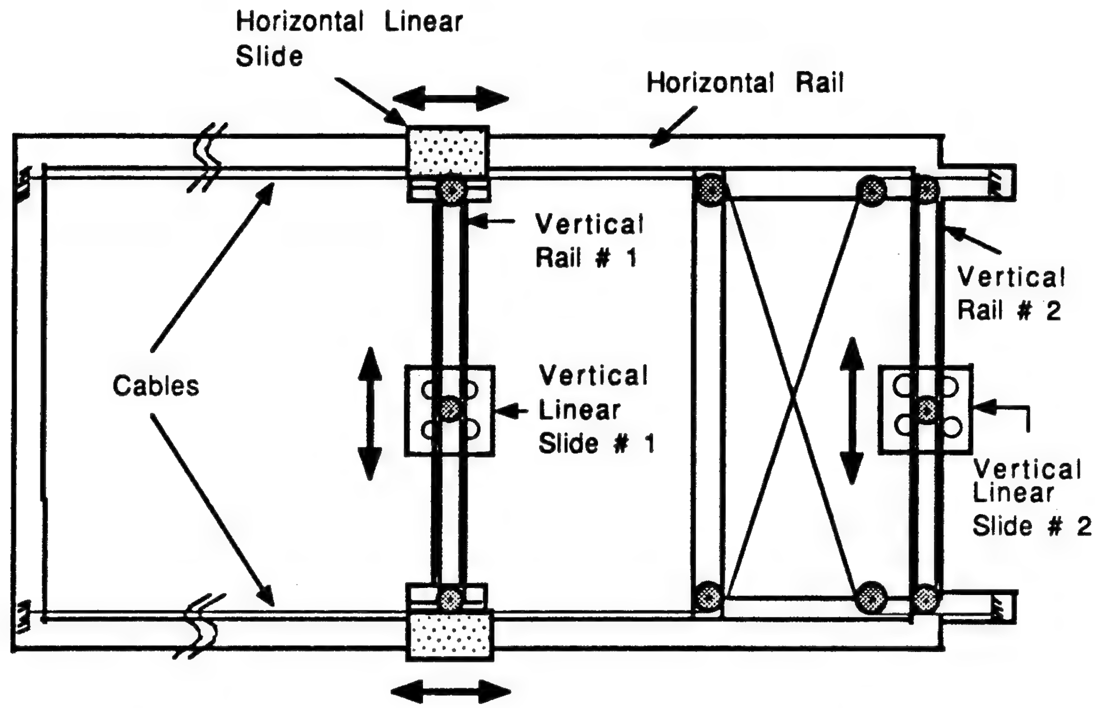

Figure 7–16: Schematic of planerizer. The inertia of the machine's vertical movement can be adjusted to be the same as the inertia of horizontal movement by adding weights on the vertical slide #2. The major possible drawback is friction in the cables and pulleys.

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7.10 Appendix A: Physical Parameters of the Monopod

Parameter	Metric Units	English Units
Lengths/angles		
Overall height	0.84 m	33 in
Overall length	1.02 m	40 in
Overall width	0.23 m	9 in
Hip height (leg fully extended)	0.74 m	29.2 in
Leg vertical travel	0.13 m	5.2 in
Leg sweep angle	±0.73 rad	±42 deg
Masses		
Total mass (body, leg, and boom)	7.9 kg	17.3 lbm
Body mass	5.4 kg	11.8 lbm
Leg mass	0.73 kg	1.61 lbm
Leg mass, unsprung	0.063 kg	0.14 lbm
Ratio of total mass to unsprung leg mass	125:1	125:1
Moments of Inertia		
Body moment of inertia (about CG)	0.22 kg-m^2	750 lbm-in^2
Leg moment of inertia (about hip)	0.097 kg-m^2	330 lbm-in^2
Ratio of body to leg moments of inertia	2.3:1	2.3:1
Performance		
Ideal no load stroke time†	0.023 s	0.023 s
Ideal no load sweep time†	0.037 s	0.037 s
Static thrust†	17.3 N	77 lb
Static hip torque†	67 N-m	590 in-lb
Ratio of static thrust to weight†	4.5:1	4.5:1
Work per Thrust Stroke†	49.9 N-m	442 in-lb
Work per Sweep Stroke†	82 N-m	724 in-lb
Leg spring stiffness	4060 N/m	23 lb/in

†Differential hydraulic pressure of 14mPa or 2000 psi.

7.11 Appendix B: Kinematics of the Monopod

Symbol	Variable Description
ϕ	Angle of the body wrt horizontal (pitch)
θ_1	Angle of foot actuator lever arm wrt leg normal
θ_2	Angle of leg wrt body normal
θ_{leg}	Angle of leg wrt vertical
θ_3	Angle of foot lever arm wrt leg normal
θ_{heel}	Angle of foot lever arm wrt horizontal
θ_{toe}	Angle of line joining ankle and toe wrt horizontal
α_1	Angle of foot actuator wrt leg
α_2	Angle of hip actuator wrt body normal
w_1	Foot actuator length, pivot to pivot
w_2	Hip actuator length, pivot to pivot
dw_1	Foot actuator displacement $(w_1 - w_{1\_0})$
dw_2	Hip actuator displacement $(w_2 - w_{2\_0})$
r_1	Effective moment arm of foot actuator about hip
r_2	Effective moment arm of hip actuator about hip
δ_f	Foot deflection $\perp$ to foot length
x	Horizontal position of the body CG
z	Height of the body CG above ground
legx	Horizontal distance from toe to body CG
legz	Vertical distance from toe to body CG

Table 7–1: Kinematic Variables of the Monopod

Figures 7–17 and 7–18 plus tables 7–1 and 7–2 define the geometric variables and constants of the monopod.

7.11.1 Leg Kinematics

The kinematic configuration of the monopod is completely determined by four measured variables: ϕ, θ_1, θ_2, and δ_f. Given these four position variables, we want to find θ_{leg}, θ_{heel}, θ_{toe}, legx, and legz. By inspection of figure 7–1, we see at once that

$$\theta_{\text{leg}} = \theta_2 + \phi. \tag{7.5}$$

From the four-bar linkage defined by $a, c, b,$ and d, we see that

$$c + a\sin\theta_1 = d + b\sin\theta_3, \tag{7.6}$$

$$\sin\theta_3 = \frac{c-d}{b} + \frac{a}{b}\sin\theta_1 \tag{7.7}$$

$$= \sin\theta_{3\_0} + \frac{a}{b}\sin\theta_1, \tag{7.8}$$

Symbol	Parameter	Metric Units	English Units
a	Foot-actuator lever arm length	0.0381 m	1.500 in
b	Foot lever arm length	0.0439 m	1.73 in
$c+h$	Leg length, hip to ankle	0.610 m	24.0 in
d	Tendon length†	0.48 m	19.0 in
e	Hip-to-body-CG offset	0.1651 m	6.500 in
f	Foot length, ankle to toe‡	0.188 m	7.4 in
g	Body half length	0.51 m	20.0 in
h	Hip to foot-actuator-lever-pivot distance	0.1016 m	4.00 in
i	Hip actuator lever length	0.0381 m	1.500 in
k	Hip to hip-actuator-pivot distance	0.1091 m	4.295 in
w_{1_0}	Foot actuator length at center of travel	0.1016 m	4.000 in
w_{2_0}	Hip actuator length at center of travel ($\theta_2 = 0$)	0.1022 m	4.025 in
θ_{3_0}	Value of θ_3 when $\theta_1 = 0$†	0.19 rad	11 deg
θ_{toe_0}	Value of θ_{toe} when $\theta_{\text{heel}} = 0$ and $\delta_f = 0$‡	0.33 rad	19 deg
α_{2_0}	Value of α_2 when $\theta_2 = 0$	1.215 rad	69.6 deg

†may change when tendon is replaced.

‡may change when foot is replaced.

Table 7–2: Kinematic Constants of the Monopod

where $\sin\theta_{3\_0} = (c-d)/b$ is the value of $\sin\theta_3$ when $\theta_1 = 0$. Therefore,

$$\theta_3 = \arcsin(\sin\theta_{3\_0} + \frac{a}{b}\sin\theta_1). \tag{7.9}$$

Also by inspection

$$\theta_{\text{heel}} = \theta_{\text{leg}} + \theta_3. \tag{7.10}$$

If we assume that foot length f is constant, and foot deflection δ_f is measured along an arc about the ankle, then we can say

$$\theta_{\text{toe}} + \frac{\delta_f}{f} \approx \theta_{\text{heel}} + \theta_{\text{toe}\_0} \tag{7.11}$$

$$= \theta_{\text{leg}} + \theta_3 + \theta_{\text{toe}\_0}. \tag{7.12}$$

Therefore

$$\theta_{\text{toe}} \approx \theta_{\text{leg}} + \theta_3 + \theta_{\text{toe}\_0} - \frac{\delta_f}{f}. \tag{7.13}$$

We find the horizontal and vertical distances from toe to hip by adding components of each of the three link lengths:

$$\text{legx} = -f\cos\theta_{\text{toe}} + (c+h)\sin\theta_{\text{leg}} + e\cos\phi, \tag{7.14}$$

$$\text{legz} = f\sin\theta_{\text{toe}} + (c+h)\cos\theta_{\text{leg}} - e\sin\phi. \tag{7.15}$$

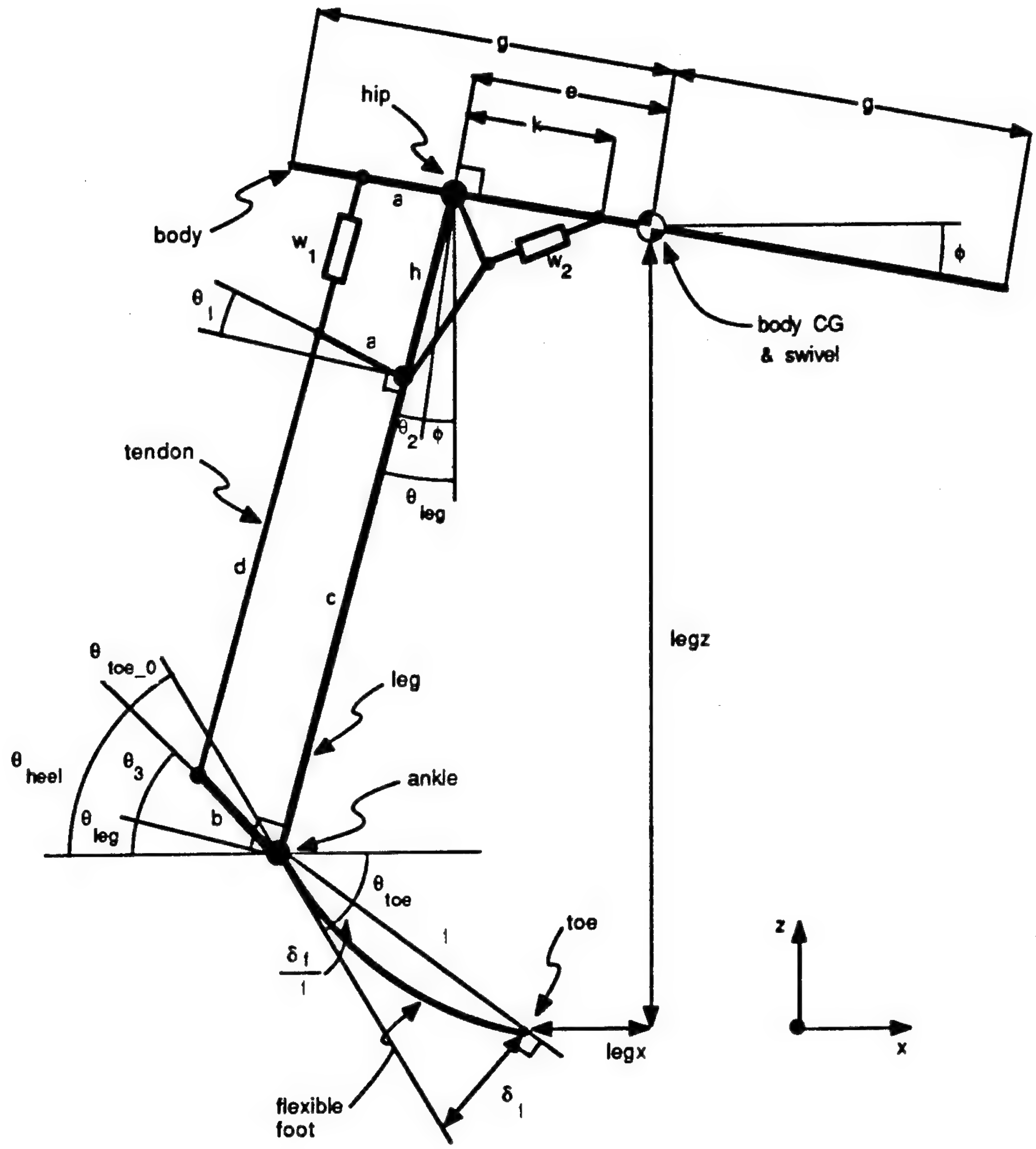

Figure 7–17: Monopod kinematics. The figure shows the overall configuration of the monopod with labels giving some of the nomenclature. Symbols are shown for the kinematic variables and parameters, which are further defined in tables 7–1 and 7–2.

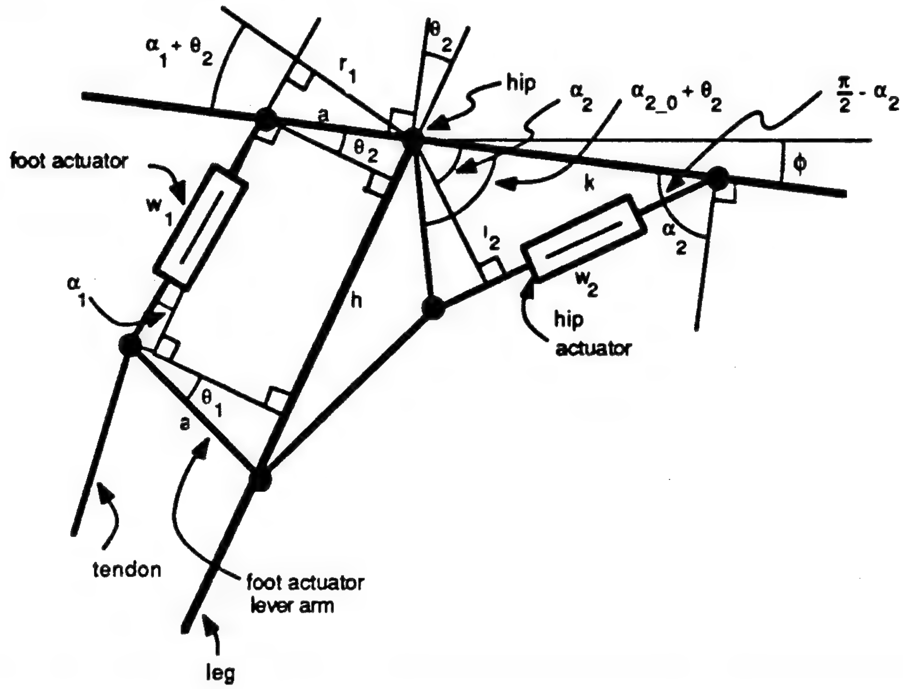

Figure 7–18: Actuator kinematics. The figure shows the geometry of the hip and actuator linkages.

7.11.2 Actuator Kinematics

Given angles θ_1 and θ_2, we want to find the actuator lengths and displacements w_1, w_2 and dw_1, dw_2, and moment arms r_1 and r_2. With reference to figure 7–18, we add length components parallel to the leg for actuator 1 to get

$$w_1 \cos\alpha_1 + a\sin\theta_2 + a\sin\theta_1 = h. \tag{7.16}$$

Therefore

$$w_1 = \frac{h - a(\sin\theta_1 + \sin\theta_2)}{\cos\alpha_1}. \tag{7.17}$$

By considering length components perpendicular to the leg we get

$$a\cos\theta_1 = w_1 \sin\alpha_1 + a\cos\theta_2 \tag{7.18}$$

or

$$w_1 = \frac{a(\cos\theta_1 - \cos\theta_2)}{\sin\alpha_1}. \tag{7.19}$$

We combine this with (7.17) to get

$$\frac{h - a(\sin\theta_1 + \sin\theta_2)}{\cos\alpha_1} = \frac{a(\cos\theta_1 - \cos\theta_2)}{\sin\alpha_1} \tag{7.20}$$

or

$$\tan\alpha_1 = \frac{\cos\theta_1 - \cos\theta_2}{h/a - \sin\theta_1 - \sin\theta_2} \approx \alpha_1 \tag{7.21}$$

The maximum possible value of α is 0.12rad (6.9 deg), so $\tan\alpha_1 \approx \alpha_1$ with less than 0.5% error. Also $\cos\alpha_1 \approx 1$ within 1%, so (7.17) becomes

$$w_1 \approx h - a(\sin\theta_1 + \sin\theta_2). \tag{7.22}$$

By design, $w_{1\_0} = h$. Therefore

$$dw_1 = w_1 - w_{1\_0} = w_1 - h \approx -a(\sin\theta_1 + \sin\theta_2). \tag{7.23}$$

From figure 7–18 we see that

$$r_1 = a\cos(\alpha_1 + \theta_2). \tag{7.24}$$

For actuator 2, we apply the law of cosines to the triangle formed by i, k and w_2:

$$w_2^2 = i^2 + k^2 - 2ik\cos(\alpha_{2\_0} + \theta_2). \tag{7.25}$$

We see also that

$$r_2 = i\cos(\alpha_{2\_0} + \theta_2 - \alpha_2) \approx i\cos\theta_2 \tag{7.26}$$

for $\alpha_2 \approx \alpha_{2\_0}$. The worst error in r_2 due to this assumption is 14% which occurs only at the limit of hip travel. If necessary, we can find α_2 and use the exact form of (7.26). Law of cosines gives

$$i^2 = k^2 + w_2^2 - kw_2\cos(\pi/2 - \alpha_2) \tag{7.27}$$

from which

$$\alpha_2 = \pi/2 - \arccos\left(\frac{k^2 + w_2^2 - i^2}{kw_2}\right). \tag{7.28}$$

7.12 Appendix C: Moment of Inertia Determination

This appendix describes the analytical and experimental techniques that have been used to determine moments of inertia of several machines and legs. The polar moment of inertia about an axis is defined as

$$J = \int_m r^2 dm \tag{7.29}$$

where

J	is the polar moment of inertia,
dm	is an infinitesimal element of mass, and
r	is the distance from the axis to the element of mass.

The moment of inertia is related to the behavior of a rigid system by Newton's Second Law, in rotational terms:

$$\mathrm{T} = J\ddot{\phi} \tag{7.30}$$

where

T	is the torque about the system center of gravity, or about a point in the system fixed to an inertial reference,
J	is the polar moment of inertia about the same point, about an axis aligned with the torque vector, and
$\ddot{\phi}$	is the angular acceleration of the system.

The parallel axis theorem arises from (7.29), and allows us to find moments of inertia about displaced parallel axes through a rigid body:

$$J_o = J_c + mr_o^2 \tag{7.31}$$

where

J_o	is the moment of inertia about a point o,
J_c	is the moment of inertia about the center of gravity,
m	is the mass of the body, and
r_o	is the displacement of point o from the center of gravity.

A planar, rigid body can be dynamically defined by three parameters: the mass, the location of the center of gravity, and the moment of inertia about the center of gravity. Its rotational behavior about any other (parallel) axis can be determined with the use of the parallel axis theorem. A three-dimensional body requires moments of inertia about three orthogonal axes through the center of gravity. Here we will consider only planar bodies, although the techniques are applicable to three-dimensional bodies as well.

The moment of inertia of a body can be determined by calculation or measurement. We have used both methods for finding moments of inertia of the leg and body subassemblies of the monopod and biped.

7.12.1 Calculation of Moments of Inertia

To find the moment of inertia of the biped's leg we assumed all individual components to be slender rods located on the leg axis. Using the parts lists and fabrication drawings, we tabulated the length, location, and mass of each component. A programmable calculator was used to determine the mass, location of the center of gravity and moment of inertia for each of the three leg subassemblies.

The biped leg comprises three subassemblies: (1) the upper leg, the parts that do not move axially; (2) the piston rod subassembly, the parts that move with the hydraulic piston and rod; and (3) the lower leg, the parts that move with the foot. We sum the masses and moments of inertia about the hip for all components of each subassembly, and locate the center of gravity relative to the hip.

We compute the moment-of-inertia contribution about the hip for each component, assumed to be a slender rod:

$$r_i = a_i + b_i/2 \tag{7.32}$$

$$J_{ci} = m_i b_i^2/12 \tag{7.33}$$

$$J_{oi} = J_{ci} + m_i r_i^2 \tag{7.34}$$

where

a_i	is the distance from the hip to the top end of the component,
b_i	is the length of the component,
r_i	is the distance from the hip to the center of gravity of the component,
m_i	is the the mass of the component,
J_{ci}	is the moment of inertia of the component about its own center of gravity,
J_{oi}	is moment of inertia of the component about the hip, and
i	identifies the component.

Then we total the contributions to obtain a total moment of inertia about the hip for a subassembly:

$$J_{ok} = \sum_i J_{oi} = \sum_i m_i b_i^2/12 + \sum_i m_i r_i^2, \tag{7.35}$$

where k denotes the subassembly. We also need the center of gravity of the subassembly, which we get by averaging first moments of mass:

$$r_k = \frac{\sum_i m_i r_i}{\sum_i m_i} \tag{7.36}$$

where r_k is the distance from the hip to the center of gravity of the subassembly. We can then find the moment of inertia of the subassembly about its own center of gravity using the parallel axis theorem:

$$J_{ck} = J_{ok} - m_k r_k^2 \tag{7.37}$$

where $m_k = \sum_i m_i$ is the mass of the subassembly k.

This technique is valid only when the system can be modeled as a group of coaxial, slender rods.

7.12.2 Measurement of Moments of Inertia

When geometric complexities, the presence of a large number of components, and/or the absence of adequate information preclude calculating moments of inertia, values can be obtained by measurement. The following technique has been found useful for our planar machines which have a low-friction pivot built into the system.

Consider a subassembly or system having its center of gravity suspended below a low-friction pivot. We can relate the moment of inertia about the pivot to the period of pendulum oscillation:

$$J_o = mgr(\tau/2\pi)^2$$

where

- J_o is the moment of inertia of the system about the pivot,
- m is the mass of the system,
- g is the gravitational acceleration,
- r is the distance from the pivot to the center of gravity of the system, and
- τ is the period of pendulum oscillation.

We can measure τ precisely by recording data from a potentiometer attached to the pivot (e.g. the pitch potentiometer), or, less desirably, by using a stopwatch. We may not know m or r precisely, but can get a good value of the product by measuring the torque needed to hold the assembly at 90 deg to its idle position:

$$\mathrm{T} = mgr$$

where T is the torque required to balance the moment generated by the weight of the system at 90 deg. We can find T by using a spring-scale or other force-measuring device at a known radius from the pivot. Care should be taken to keep the scale vertical, and the system at 90 deg to its idle position.

We used this method to find the moment of inertia of the biped (body plus legs) about the pivot by wiring the legs to the body so the whole system behaved as a rigid body. We then found the moment of inertia of the body alone by subtracting the moment of inertia of the two legs about the pivot.

This method was also used to measure the leg moment of inertia of the monopod. To do this we disconnected both hydraulic actuators so the leg could swing freely. We wired the foot in the desired position.

Chapter 8

Passive Dynamic Running

8.1 Introduction

Running is a motion that combines a vertical oscillation of the body with a fore-aft oscillation of the legs. Previous work by us and by others has considered how elastic energy storage can be used to generate vertical motion of the body, without requiring a large expenditure of energy on each step. The body can bounce on springy legs during the stance phase, storing a portion of the kinetic energy as strain in the leg springs, and releasing it later to help power the next step. This approach is appealing because it offers energetically efficient vertical motions of the body, and contributes to simplified control. This approach is used by some legged robots (Raibert 1986) and many animals (Alexander 1988).

In this paper we consider how elastic energy storage might also be used to generate the fore-aft oscillations of the legs, without large energy expenditure. The goal is to avoid losing the kinetic energy of the legs each time they reverse their fore-aft sweeping motion. Such losses are particularly severe at high running speeds. Our approach is to turn the legs into harmonic oscillators which approximate the motions needed for running. The legs can be made into harmonic oscillators by introducing torsional springs at the hip joints. The resulting leg oscillations move the foot backward with respect to the hip during the stance phase, and forward in preparation for the next step during the flight phase.

To get a stable running pattern, the motions of the body and the legs must be coordinated. The hip oscillation must be coordinated with the:

- forward travel of the body during the stance phase so the foot does not slip on the ground.
- pitching of the body, so that the body pitch angle reverses sign once during stance and once during flight.
- vertical bounce of the body, so the system balances.
- vertical flight of the body, so the foot is correctly positioned on the next step.

The objective of this study was to see if we could design a simple passive system that could run with trajectories satisfying these constraints. We implemented computer simulations of a planar one-legged model composed entirely of springs, masses, and linkages. The model is shown in Figure 8–1. We manipulated the running trajectories by tuning the natural frequency of the vertical bouncing motion to be a specific fraction of the natural frequency of the leg swinging oscillation, and by choosing initial conditions according to the running speed. We manipulated the parameters until phase plots of several variables indicated reentrant behavior. The observed running trajectories had a high degree of reentrance, but in no case were they perfectly reentrant and lossless.

The systems we consider are passive in that they are made up of springs, links, and masses, with no actuators or other source of external energy. In doing these studies we do not suggest that physical legged systems can operate passively for sustained periods of time. A source of energy is needed to make up for mechanical losses, some of which are unavoidable, and a source of control is needed to maintain the reentrant running trajectory. Once the passive part of the system is understood, it should be possible to introduce actuators and algorithms that provide energy and control. We expect physical legged systems that use this approach to have a *tuned gait*, for which the energy efficiency will be highest. At other gaits the system will perform with reduced efficiency, depending on how far the gait deviates from the tuned gait.

8.2 Background

Means for providing efficient fore-aft motion of the legs have been considered in previous work. Mochon and McMahon (1980, 1981) modeled the human leg as a compound pendulum. They showed that the behavior of the leg during the swing phase of human walking could be accomplished as a passive ballistic motion requiring no energy other than that delivered through forward motion of the hip. McGeer (1989a) built a simple nearly passive machine that walks using ballistic swing motions similar to those modeled by Machon and McMahon's. His machine, which has been demonstrated in the laboratory, uses gravity to sustain the walking motion. More recently he has analyzed a two-legged passive dynamic running model and given conditions for reentrant behavior and stability (McGeer, 1989b).

Ivan Sutherland discussed a *tuning fork* model of locomotion in 1983 (Sutherland 1983). He noticed that the motions exhibited by the tines of a tuning fork were somewhat like the leg motions used by animals during walking and running.

Alexander (1988) has done a broad set of investigations that ask how springs are used in animal locomotion. He has proposed that the aponeurosis, a sheet of tendon found in the backs of some quadrupeds, might be act as a tension spring when the back is bent, and the vertebral column might act as a compression spring. These springs could reverse the direction of the legs during the gathered phase of galloping. He estimates that about half of the internal energy could be stored in the aponeurosis and and vertebral column of a fast galloping deer (Alexander *et al.* 1985).

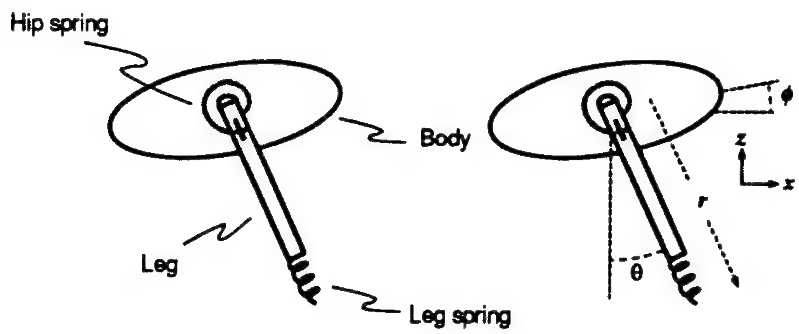

Figure 8–1: Diagram of planar one-legged model used in simulations.

8.3 Models

To study passive dynamic running we used a computer simulation of a planar one-legged model. The model is shown in figure 8–1. The model has a body of mass m_b and moment of inertia J_b measured about the hip, and a leg of mass m_ℓ and moment of inertia J_ℓ, also measured about the hip. The model has two springs. The hip spring acts between the leg and the body, exerting torque about the hip axis. The hip spring has stiffness k_h. A leg spring acts along the leg axis, between the lower part of the leg and the support surface. The leg spring is massless, exerts force only during the stance phase, and has stiffness k_ℓ.

There is a third spring that acts tangent to the leg axis. This spring, stiffness k_t, represents the combined lateral compliance of the foot and the ground. The constitutive relations for the leg and tangent springs determine the forces applied to the foot during contact. The equations of motion that describe the system are:

$$\ddot{x}(m_b + m_l) = F_x \tag{8.1}$$

$$\ddot{z}(m_b + m_l) = F_z - (m_b + m_l)g \tag{8.2}$$

$$\ddot{\phi}J_b = k_h(\theta - \phi) \tag{8.3}$$

$$\ddot{\theta}J_\ell = r(F_x \cos\theta + F_z \sin\theta) - k_h(\theta - \phi) \tag{8.4}$$

A variable-step Runge-Kutta routine was used to integrate the equations of motion to obtain behavior as a function of time. For each simulation, we chose initial conditions and adjusted parameters to get desired behavior. In the following paragraphs we describe the behavior of the model when it runs, and the methods we used for choosing and modifying initial conditions.

Symbol	Description	Nominal Value
g	acceleration of gravity	$9.81\,\mathrm{m/sec^2}$
m_b	body mass	$10.0\,\mathrm{kg}$
J_b	body moment of inertia	$2.5\,\mathrm{kg\cdot m^2}$
k_h	hip spring constant	varies
r	leg length	$0.7\,\mathrm{m}$
r_0	leg zero-force length	$0.7\,\mathrm{m}$
m_ℓ	leg mass	$1.0\,\mathrm{kg}$
J_ℓ	leg moment of inertia	$0.25\,\mathrm{kg\cdot m^2}$
k_ℓ	leg spring constant	varies
k_t	leg tangent spring constant	$k_\ell/10$
x	forward position of hip	state
z	vertical position of hip	state
θ_b	body pitch angle w.r.t horizontal	state
θ_ℓ	leg angle w.r.t. vertical	state
θ_0	hip spring zero position	$0.0\,\mathrm{rad}$
$\dot{z}_{lo}$	vertical liftoff velocity	state
F_x	horizontal ground force	output
F_z	vertical ground force	output
$(.)_o$	initial condition for (.)	output

Table 8–1: Parameters of One-Legged Passive Model

Vertical Bouncing

During flight, the center of mass travels a parabolic trajectory determined by the vertical position and velocity at liftoff

$$z(t) = z_{lo} + \dot{z}_{lo}t - \frac{gt^2}{2} \tag{8.5}$$

where z_{lo}, $\dot{z}_{lo}$ are the vertical position and velocity at liftoff, and g is the acceleration of gravity. The peak altitude is given by

$$z_{max} = \frac{\dot{z}_{lo}^2}{2g}. \tag{8.6}$$

The duration of the flight phase is

$$T_f = 2\sqrt{\frac{2z_{max}}{g}}. \tag{8.7}$$

During stance, the vertical motion is a harmonic rebound determined by the system mass bouncing on the leg spring. The natural frequency of this rebound is

$$\omega_\ell = \sqrt{\frac{k_\ell}{m_b + m_\ell}} \tag{8.8}$$

From McMahon and Cheng (1989) we know that the duration of the stance phase in vertical hopping is

$$T_s = \frac{2(\pi - \arctan(|\dot{z}_{lo}|\omega_\ell/g))}{\omega_\ell} \tag{8.9}$$

In this paper we approximate the stance phase as one half cycle of the natural oscillation. In this case

$$T_s \approx \frac{\pi}{\omega_\ell}. \tag{8.10}$$

This approximation is valid when the vertical velocity at touch down is large compared to g/ω_ℓ, or when the ratio of flight duration to stance duration is greater than 1. This analysis of the vertical motion is strictly valid only for hopping in place, without forward travel and sweeping motions of the legs. Closed form solutions for the stance duration of non-vertical hopping are not known (McMahon and Cheng, 1989).

Hip Oscillation

The natural frequency of the hip oscillation is given by

$$\omega_h = \sqrt{\frac{k_h}{J_{eff}}} \tag{8.11}$$

where k_h is the hip spring constant and $J_{eff} = J_b J_\ell/(J_b + J_\ell)$ is the effective moment of inertia of the combined leg and body about the hip. The characteristic period of the hip oscillation is given by

$$T_h = \frac{2\pi}{\omega_h}. \tag{8.12}$$

The horizontal displacement of the foot from the hip is

$$x_f = r\sin(\theta_{max}\sin(\omega_h t)) \tag{8.13}$$

where r is leg length, θ is the leg angle with respect to the vertical, and θ_{max} is the amplitude of the hip oscillation. We have assumed that the hip spring is at rest when $\theta = \phi = 0$. This function, plotted in figure 8–2, is approximately linear in time for small values of time and θ_{max}.

8.4 Choosing Parameters for Passive Dynamic Running

Figure 8–2 plots foot motion for an ideal system traveling forward at constant speed. During the stance phase, the foot does not move with respect to the ground, so the velocity of the foot with respect to the body is the negative of the body's forward velocity. Therefore, for forward travel at constant speed, foot position with respect to the body is a linear function of time. During the flight phase, the only constraint on foot motion is that it move forward in time for the next stance phase.

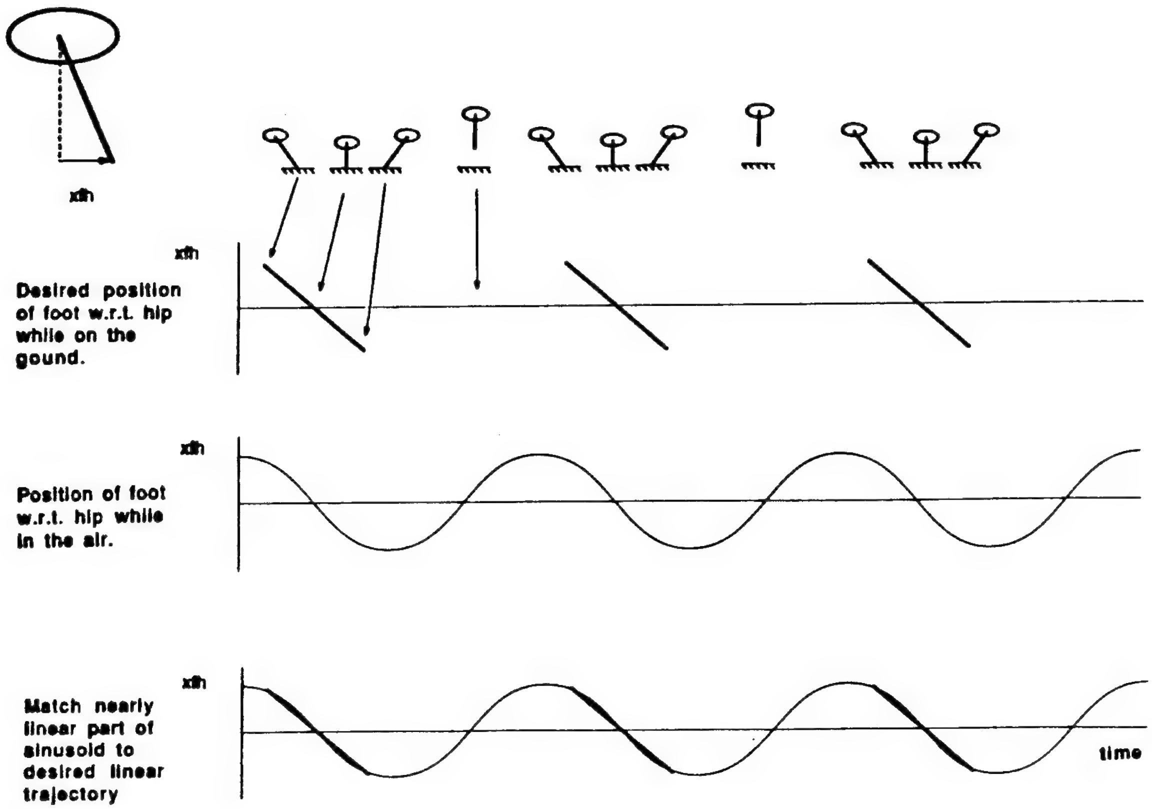

Figure 8-2: Harmonic oscillation of the hip moves foot approximately as desired for constant speed forward running. A) Plot of horizontal position of the foot with respect to the hip during the stance phase, for constant speed forward travel. B) Plot of horizontal position of foot with respect to the hip for harmonic hip oscillation, assuming a fixed leg length. C) Comparison of the two curves shows that harmonic hip motion would provide a good approximation to constant speed forward travel.

Our basic approach to finding passive reentrant running trajectories is based on the fact that for suitable parameter values, harmonic hip motion generates foot positions as a function of time that closely approximate the foot motions found in constant speed forward travel. Figure 8-2 shows how foot motion produced by a harmonic hip oscillation can be tuned to approximate the foot motions needed for ideal constant speed travel. The approximation is based on the linearity of the sin function for small values of its argument.

The remainder of this section describes how we chose system parameters and initial conditions to find reentrant trajectories for the model. We assumed fixed values of body mass m_b, body moment of inertia J_b, leg mass m_ℓ, leg moment of inertia J_ℓ, and nominal leg length r_0. Given a desired running speed $\dot{x}_d$ and step period T_{step}, we chose the spring constant for the hip k_h, spring constant for the leg k_ℓ, initial leg pitch rate $\dot{\theta}_0$, initial body pitch rate $\dot{\phi}_0$, and initial body altitude z_{max}.

Hip spring constant k_h

The stiffness of the hip is chosen so the hip undergoes one complete oscillation during one complete step. The natural frequency of the hip oscillation is

$$\omega_h = \frac{2\pi}{T_{step}} = \sqrt{\frac{k_h}{J_{eff}}}. \tag{8.14}$$

From (8.14) we see that

$$k_h = (2\pi/T_{step})^2 J_{eff}. \tag{8.15}$$

Leg spring constant k_ℓ

The stiffness of the leg is chosen to establish the duration of the stance phase. We define the duration of the stance phase as a duty factor ρ, which expresses the duration of the stance phase as a fraction of the step period

$$\rho = T_s/T_{step} = \frac{\omega_h}{2\omega_\ell}, \tag{8.16}$$

assuming the stance phase is one half cycle of leg spring oscillation. We chose this duty factor so that the stance phase occurs during the roughly linear portion of the foot's fore-aft travel. Small values of ρ give the best linearity, but result in larger ground forces and longer flight durations. We experimented with values of ρ between 0.125 and 0.35. A good value for smaller the value for ρ is 0.125, but the smaller the better.

Given the natural frequency of the hip and a value of ρ, the leg spring constant k_ℓ can be found

$$k_\ell = (\pi/\rho T_{step})^2 m. \tag{8.17}$$

Because we do not have an exact expression for T_s, (8.17) gives a value for k_ℓ that only approximates the desired value for ρ. The precisely desired value of ρ is obtained by adjusting k_ℓ through a series of trials.

Initial values for leg angular rate $\dot{\theta}$ and body angular rate $\dot{\phi}$

The amplitude of the hip oscillation and the speed of the foot as it moves back and forth are both determined by the initial value of the hip rotation rate, $\dot{\theta}$. This parameter is selected so the speed of the foot moving backward during the stance phase matches the desired forward speed of the body.

The simulation is begun by dropping the model from a specified height. Therefore, the initial conditions for the simulation specify the state of the system at mid flight. The angular rate of the hip at mid flight equals the angular rate at mid stance, with a sign reversal. The hip rate is chosen to match the backward foot velocity at mid stance to the desired forward speed

$$\dot{\theta}_0 = \dot{x}/r \tag{8.18}$$

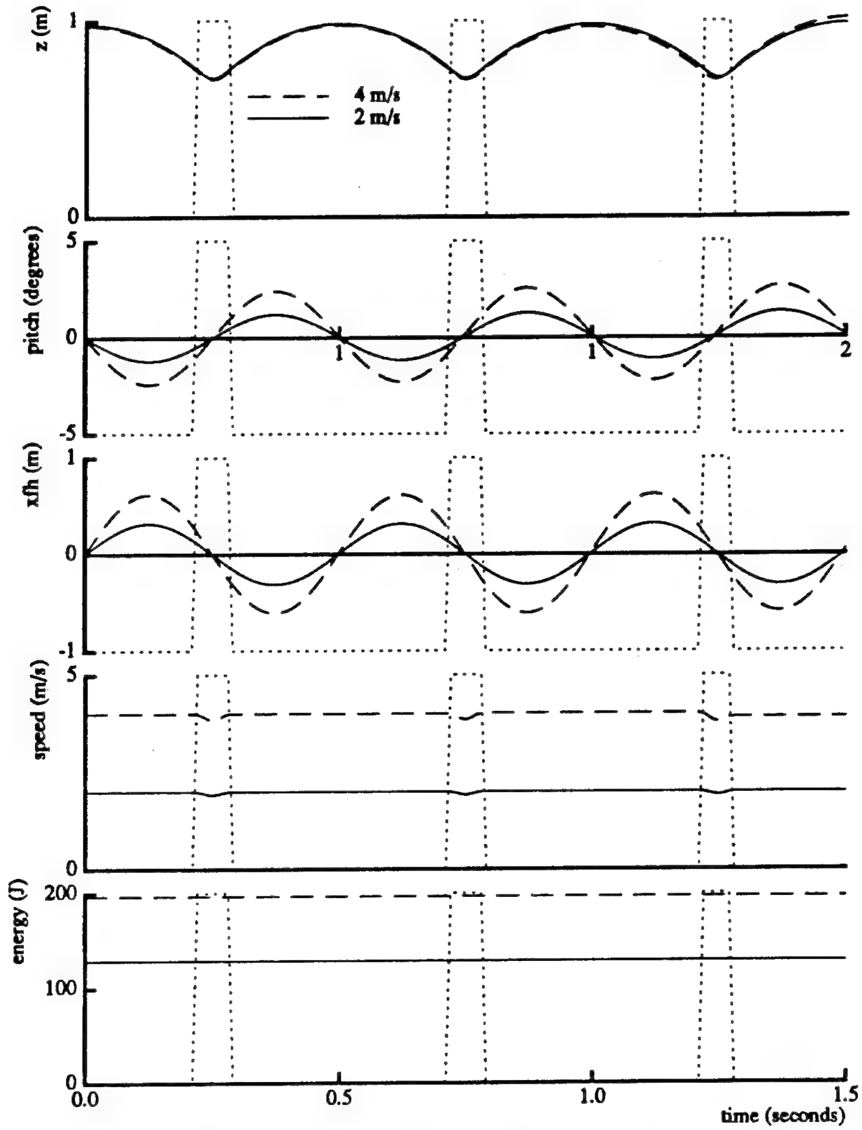

Figure 8–3: Passive dynamic running of planar one-legged hopper. Data are from computer simulations of model shown in figure 8–1a and described in table 8–1. Data are shown for running speeds of 2 and 4 m/s. Vertical dotted lines bracket the stance phase. Solid) $\dot{x} = 2\,\mathrm{m/s}$, $k_h = 35.9\,\mathrm{kg \cdot m}$, $k_\ell = 29{,}750\,\mathrm{kg/m}$, $z_{max} = 0.98\,\mathrm{m}$, $\dot{\theta}_0 = 2.67\,\mathrm{rad/sec}$. Dashed) $\dot{x} = 4\,\mathrm{m/s}$, $k_h = 35.9\,\mathrm{kg \cdot m}$, $k_\ell = 25{,}500\,\mathrm{kg/m}$, $z_{max} = 0.97\,\mathrm{m}$, $\dot{\theta}_0 = 5.33\,\mathrm{rad/sec}$.

From (8.18) we see that the maximum leg angle is $\theta_{max} = \dot{\theta}_0/\omega_h$.

To maintain zero angular momentum during flight, the body and leg must counteroscillate with rates and amplitudes inversely related to their moment of inertia

$$\frac{\dot{\phi}}{J_\ell} = -\frac{\dot{\theta}}{J_b}. \tag{8.19}$$

The initial value of body pitch rate is therefore $\dot{\phi}_0 = (\dot{\theta}_0 J_\ell)/J_b$.

Initial value for vertical position of body, z_{max}

The duration of the flight phase is manipulated by choosing the initial altitude of the body. The duration of a step is $T_{step} = T_s + T_f$. We chose the parameters of the system so that $T_s = \rho T_{step}$, which gives a flight phase duration

$$T_f = (1 - \rho) T_{step}. \tag{8.20}$$

The initial altitude that provides the correct flight duration is

$$z_{max} = \frac{g}{8}(1-\rho)^2 T_{step}^2 + \frac{r_0 \dot{\theta}_0}{\omega_h} \cos \frac{\omega_h T_s}{2} \tag{8.21}$$

where the second term is the altitude of the body at touchdown. Examining (8.14) through (8.21), we see that only three parameters are required to specify the motion: T_{step}, ρ, and $\dot{x}$.

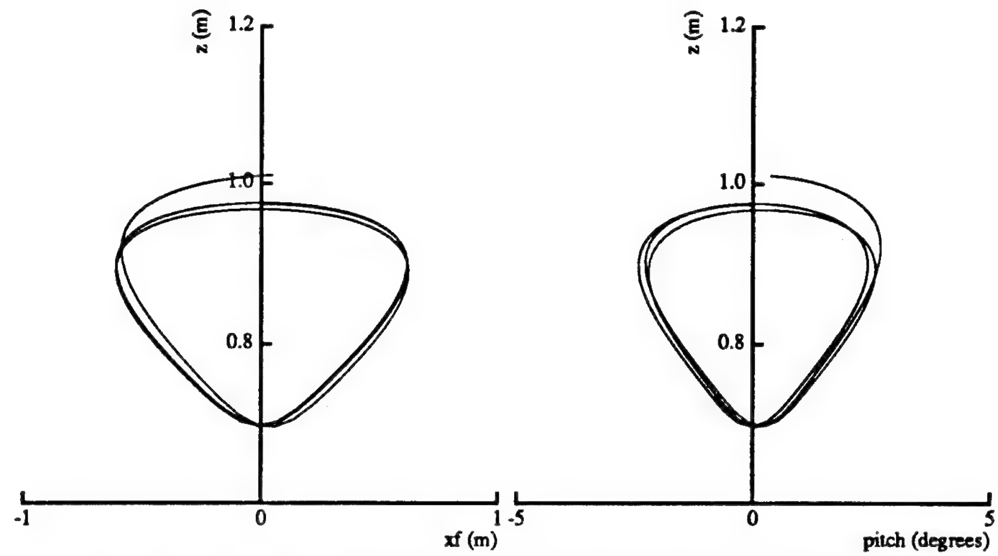

Figure 8–4: Data from figure 8–3 (4 m/s) replotted as one variable against another. If the behavior were perfectly reentrant, the plotted trajectories would perfectly superimpose. Both plots are for the same run of the one-legged model. $\dot{x} = 4\,\text{m/s}$, $k_h = 35.9\,\text{kg}\cdot\text{m}$, $k_\ell = 25,500\,\text{kg/m}$, $z_{max} = 0.97\,\text{m}$, $\dot{\theta}_0 = 5.33\,\text{rad/sec}$.

8.5 Results

Figures 8–3 and 8–4 show the results of a typical simulation of the model. The initial conditions used to generate the figures were determined from equations (8.5) – (8.5) developed above, and through a hand optimization process. During optimization, the leg spring constant was changed until we obtained the required stance duration and a reentrant running cycle.

Once the hip and leg spring stiffnesses are chosen it is possible to manipulate the initial conditions to run at different speeds. Figure 8–3 includes data for two running speeds. The physical parameters of the model were the same for both running speeds, with adjustments made only in the initial conditions. This suggests that a single machine could run at a range of speeds, without requiring mechanical tuning.

Figure 8–5 shows data for simulations with two different ratios of leg moment of inertia to body moment of inertia. The only variable affected by this change is the magnitude and rate of body pitching. All other variables remain unchanged.

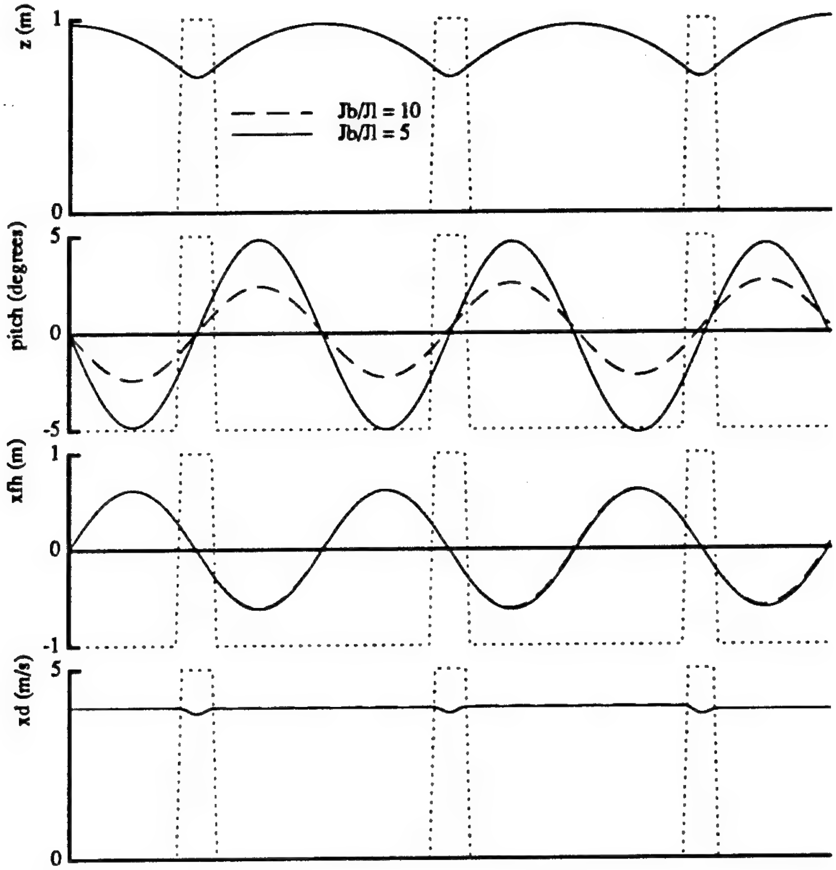

Figure 8–5: The ratio of body moment of inertia to leg moment of inertia determines the amount of body pitching. A larger body moment of inertia results in smaller body pitch amplitude. In all other respects, the two running motions are indistinguishable. $\dot{x} = 4\,\mathrm{m/s}$, $k_h = 65.8\,\mathrm{kg \cdot m}$, $k_\ell = 25,500\,\mathrm{kg/m}$, $z_{max} = 0.97\,\mathrm{m}$, $\dot{\theta}_0 = 5.33\,\mathrm{rad/sec}$.

The trajectories explored in this paper represent unstable equilibria. Although the behavior is reentrant if the system is undisturbed, there is no mechanism to return the system to the trajectory if there is a disturbance that causes it to deviate. This is seen more clearly in figure 8–4, where the position of the foot with respect to the hip and pitch angle are plotted against altitude. These data show a gradual shift in the phase trajectory. If the trajectory were stable, it would return to the equilibrium limit cycle. A complete implementation would include a control mechanism to eliminate this sort of drift.

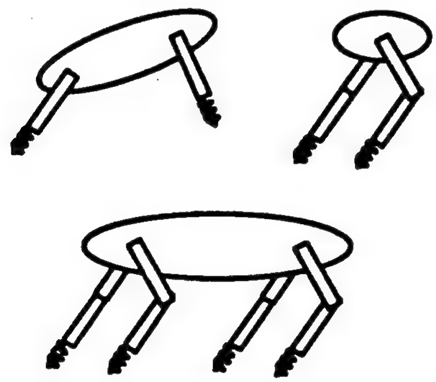

Figure 8–6: Models with compound pendulum legs and springy hips. The leg would fold during the swing phase, and pogo during the stance phase.

So far we have considered passive dynamic running in the context of the planar one-legged hopping machine with a telescoping leg, as described in this paper. We have also considered a planar two-legged system with telescoping legs, as shown at the left of the middle line in figure 8–6. So far, we have found reentrant trajectories for this planar two-legged system when the legs are used in synchrony. We would like to find a passive bounding trajectory that uses the legs out of phase, but have not yet done so. Our plans also include considering systems with compound pendulum legs, like those studied by Mochon and McMahon (1980) and shown in figure 8–6.

8.6 References

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Chapter 9

Internal Combustion Actuation

9.1 Introduction

We are exploring a new kind of actuator to use in systems that are self-contained and must carry their own power. A conventional self-contained power system, like the one shown in figure 9–1a, incorporates a chain of energy conversions and transfers that intermediates between combustion of a fluid fuel such as gasoline, and the controlled delivery of force and power to the mechanism. The present idea is to improve the power to weight ratio of the overall system by eliminating links in the conversion chain. The proposed system, shown in figure 9–1b, will combust fuel directly in the actuator, thereby eliminating engine, shafts, couplings, and hydraulic pump. The fuel and fuel supply tank remain unchanged. The carburetor and servovalve merge into a single fuel-valving mechanism. The engine, and hydraulic pump are eliminated.

The actuator is similar in form to conventional actuators, but it must dissipate heat and match the impedance of the exploding fuel to the driving load of the output linkage. Additional valves may be required to exhaust hot gasses at high rate. Techniques for providing fuel and exhaust valving are at the heart of the proposed development. Initial designs will perform these functions using conventional mechanical and electromagnetic valves, including servovalves. In future designs it may be necessary and possible to use specially designed micro fuel injectors fabricated with integrated technology. Computing will play a role in providing suitable operation of fuel and exhaust valves.

The advantages of the internal combustion actuator over conventional designs would be its high power to size and power to weight ratios, and the simplicity gained from eliminating major sets of moving parts. Improved fuel efficiency might also be possible at some scales, although probably not at large scale.

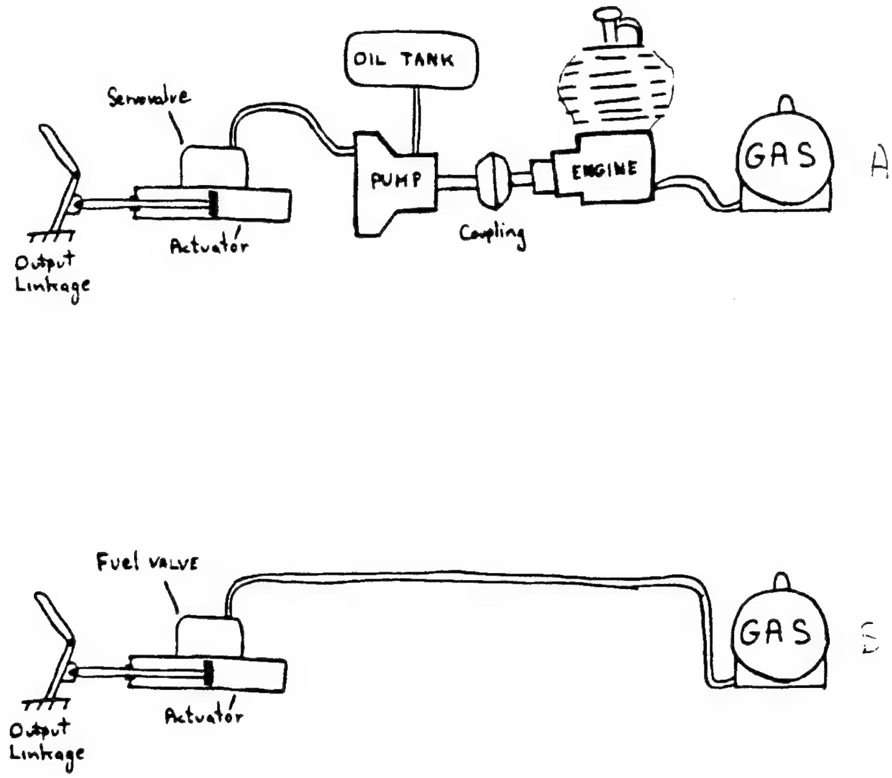

Figure 9–1: A) A conventional power system employing an internal combustion engine, a hydraulic pump, and hydraulic servoactuators. B) An *internal combustion actuator*. The engine, coupling linkages, and hydraulic pump are eliminated, since the fuel is combusted directly in the linear actuator.

9.1.1 Direct Actuation

The idea behind the internal combustion actuator is to avoid a long chain of energy conversions, each of which reduces efficiency and adds weight. There is an increasingly direct connection between combustion and propulsion as one moves down the following list:

- diesel train, steam ship
- automobile
- propeller aircraft
- jet aircraft
- rocket

The coupling between combustion and thrust is quite tight in the jet aircraft and rocket, though both rely on conventional actuation to move control surfaces. Space systems with

reaction-jet attitude control are the only examples I know of that use internal combustion actuation for motion control. We are left with a central question of this study: Can the extremely high energy density and rapid response of combustible fluid fuels such as gasoline be harnessed to produce controlled motion without a lot of mechanical overhead?

9.1.2 Use of Internal Combustion Actuators in Running Machines

We are exploring the internal combustion actuator in the context of legged locomotion. The application to legged robots is ideal because it requires self-contained power in order to be practical, and there is a high penalty for size and weight. Legged systems are a particularly good application for internal combustion actuators because they use rhythmic motions, which should be simpler to control with the proposed device than the discrete motions found in manipulation. Internal combustion actuators may eventually be useful in other self-contained applications that use servoactuators.

Internal combustion actuators are suited for two functions in legged locomotion. One function is to power the vertical thrust that each leg delivers to make the machine rebound from the ground†. A second function is to power the fore and aft sweeping motions of the legs. The vertical motion is particularly good for internal combustion actuation, because each vertical thrust is preceded by a compression phase, in which the legs shorten under the weight of the body. This compression phase can be used to prepare for actuation as does the compression stroke in a conventional internal combustion engine. Some other means will be required to provide compression for the sweeping motions of the legs, especially to get started. Pairs of actuators, return springs, and temporary gas storage elements are possibilities. For both the vertical and sweeping motions of the legs, springs in series with actuators will be required to match impedances.

In evaluating the internal combustion actuators, both on the bench and in the context of running machines, we expect to reach one of the following conclusions:

- Internal combustion actuators are totally unworkable with existing or near term technology.
- Internal combustion actuators can be used for main power delivery, but control must be exercised through conventional actuators.
- Internal combustion actuators can provide a crude control at low bandwidth, but they must be used in conjunction with a low power/high-bandwidth actuation system for precise motions.
- Internal combustion actuators can deliver controlled output at high bandwidth.

This list of possibilities comes from the expectation that internal combustion actuation will be easy when the required motion is repetitive, but more difficult when the required

† A gasoline powered pogo-stick using internal combustion actuation to power the vertical motion was sold through magazines in the early 1970's. That design suffered from a poor match between the rate of vertical acceleration of the rider and the duration of the vertical thrust, and because there was no means for controlling the magnitude of the combustion.

output of force, amplitude, and rate vary from motion to motion and within each motion. Another likely possibility is that we can trade some of the simplicity shown in figure 9–1b for improved control.

9.2 Progress on the IC-Actuator

We have built a series of four bench-top hydrogen engines that have allowed us to experiment with various overall configurations, fuel/air mixtures, compression arrangements, seals, and instrumentation.

We chose to use hydrogen as the fuel for these experiments because it has a higher energy mass density (142 MJ/kg) than propane (50 MJ/kg) or gasoline 47 MJ/kg. Hydrogen is also convenient because the principal combustion byproduct is water vapor, permitting us to run indoor experiments without special exhaust handling equipment. Hydrogen is also safer to store in the laboratory than either gasoline or propane.

9.2.1 One Sided Test Actuator

We built a one-sided internal combustion actuator and an actuator system controller, to determine the linkage between system input variables such as fuel/air ratio and precombustion pressure preload, and system output.

The one sided actuator shown by figure 9–2 consists of:

1. A brass power piston tightly fitted into the bore of a thick-walled aluminum cylinder head. The small annular gap (0.001 inch) between the piston and the cylinder acts as a dynamic seal to protect the O-ring riding at the top of the piston from the hot combustion gasses.

2. A manifold block to carry the fuel/air mixture to the cylinder and to mechanically support the other assemblies.

3. A fuel/air mix chamber that acts as a carburetor and allows precise control of the ratio of air and hydrogen. The chamber holds enough fuel/air mixture charge for 6 piston firings.

4. A simple sensor and control system consisting of a high resolution pressure sensor (0.01 psi) in the carburetor, a high pressure (5000 psi) pressure sensor in the combustion chamber and a single board computer as a local controller. The processor was a Motorola 6811 micro-controller.

5. A glow plug was used for detonation.

6. A high-pressure stainless steel check valve and a single failsafe normally closed commercially purchased external charge injections valve protected the input fuel line.

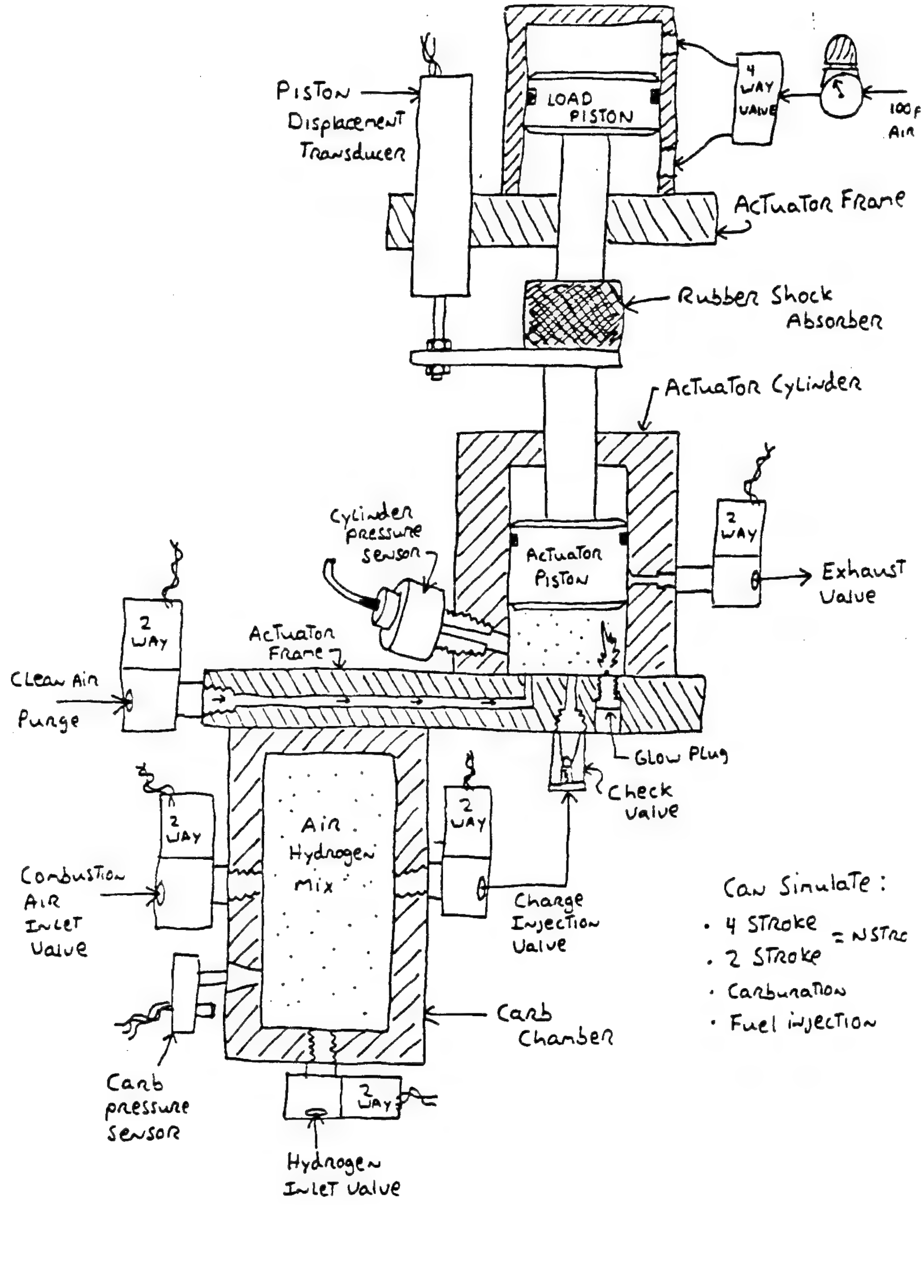

Figure 9–2: One-sided internal combustion test bench.

7. A pressure regulateable double acting air cylinder was attached to the actuator piston through a shock absorbing coupling to both position the actuator piston during the motoring parts of a cycle, and to act as a load during the firing parts of a cycle.

8. The control program performed all of the cycle sequence and experimental data recording functions automatically.

We installed O-ring seals on the piston and at all juncture points and used commercial high pressure valving wherever possible. The fixture holds pressure for a few minutes. The operation of the test station varied depending on which firing cycle was under investigation. When operated with a four stroke cycle, the controller executes the following steps:

1. Charge the carburetor to the correct mix.

2. Void the piston chamber by drawing the piston down with the exhaust valve open.

3. Induct a fresh explosive charge by closing the exhaust valve opening the injection valve and driving the piston up, creating a partial vacuum that pulls in the charge from the carburetor.

4. Compress the charge by closing all valves and driving the piston down. The more the charge is compressed, the more explosive it becomes because of the proximity between fuel and oxidant.

5. Fire by turning on the glowplug at a preset pressure level.

6. Record data during the power stroke, as the hot expanding gasses formed by the explosion push against the actuator piston which in turn, is loaded by the air cylinder. The pressure rise in the cylinder caused by the interaction of the two is recorded from the onset of fuse activation for about 1.2 sec. During this interval 512 cylinder pressure and piston displacement data samples are recorded.

7. Open the exhaust valve after the last data point is recorded, releasing the cylinder gasses. Because of the low frequency of firing, the actuator body never gets hot enough to keep the combustion byproducts (in this case water vapor) in a gaseous form. As the vapor hits the cold cylinder walls it condenses and a significant fouling problem occurs. Clean air at 100 psi is injected through a purge valve to blow out any accumulation of liquid between cycles. At higher firing rates, the vapor stays vapor and steam can be seen issuing from the exhaust valve.

Results

The internal combustion test station has 5 major parameters that can be experimentally varied to study its feasibility for use as a controllable actuator:

1. Air/fuel mix ratio, which controls force generated

2. Compression ratio, which controls energy return for energy invested

3. Type of combustion cycle, 0, 2, or 4 strokes

4. Frequency of firing

5. Type of load applied to piston

Experiment I: Effect of Air/Fuel Mix

Hydrogen is a very flexible fuel to use for internal combustion actuation because it is explosive over a wider range of fuel/air mixture ratios than any other fuel. To get an idea of how force output could be controlled by selectivity altering this ratio, a series of experiments were run in which the ratio was varied from 10% hydrogen to 90% hydrogen air with the explosion pressure profile measured for each one. These ratios were those present in the carburetor and not those present in the cylinder. The ratio in the cylinder was probably considerably leaner because of mixing with air entrained in the manifold block passages. All other variables were held constant. With a ration of 10% (figure 9–3 top left) the pressure time curve shows a slight pressure rise caused by the piston compressing the fuel/air mixture, but no explosion takes place. With a mixture ratio of 40% (figure 9–3 top right) an explosive peak that followed ignition was followed by a pressure rise that could be attributed to hot expanding gasses. With a ratio of 60% (figure 9–3 bottom left) the explosive peak is followed by a pressure rise that almost triples the pressure developed by compression of the charge alone. The surplus pressure (the area under the 60% curve and above the 20% curve) is excess energy capable of doing useful work (see figure 9–3 bottom right).

The complete family of explosion curves shows a symmetry of form that rises from no explosion at 20% to a maximum at 60% and then down to no explosion again at 80%. Varying other parameters causes the family of curves to shift and distort in unpredictable ways. However, for a given set of parameters, the actuators force output might be controlled by varying the mix ratio. This would require the development of a precise high-speed mixing system if the actuator were to operate at high frequencies.

Experiment 2

The compression ratio relates the volume of the cylinder when it is first filled with charge to the volume of the cylinder when the charge is ignited. If the piston does not move the ratio is 1:1. If the piston moves halfway down the cylinder the ratio is 2:1, etc. Commercial engines operate with compression ratios between 8:1 and 12:1. At low compression ratios the molecules of fuel and oxidant are relatively far apart and tend to burn rather than explode. At high compression ratios the molecules are crowded together and combustion takes place almost instantaneously.

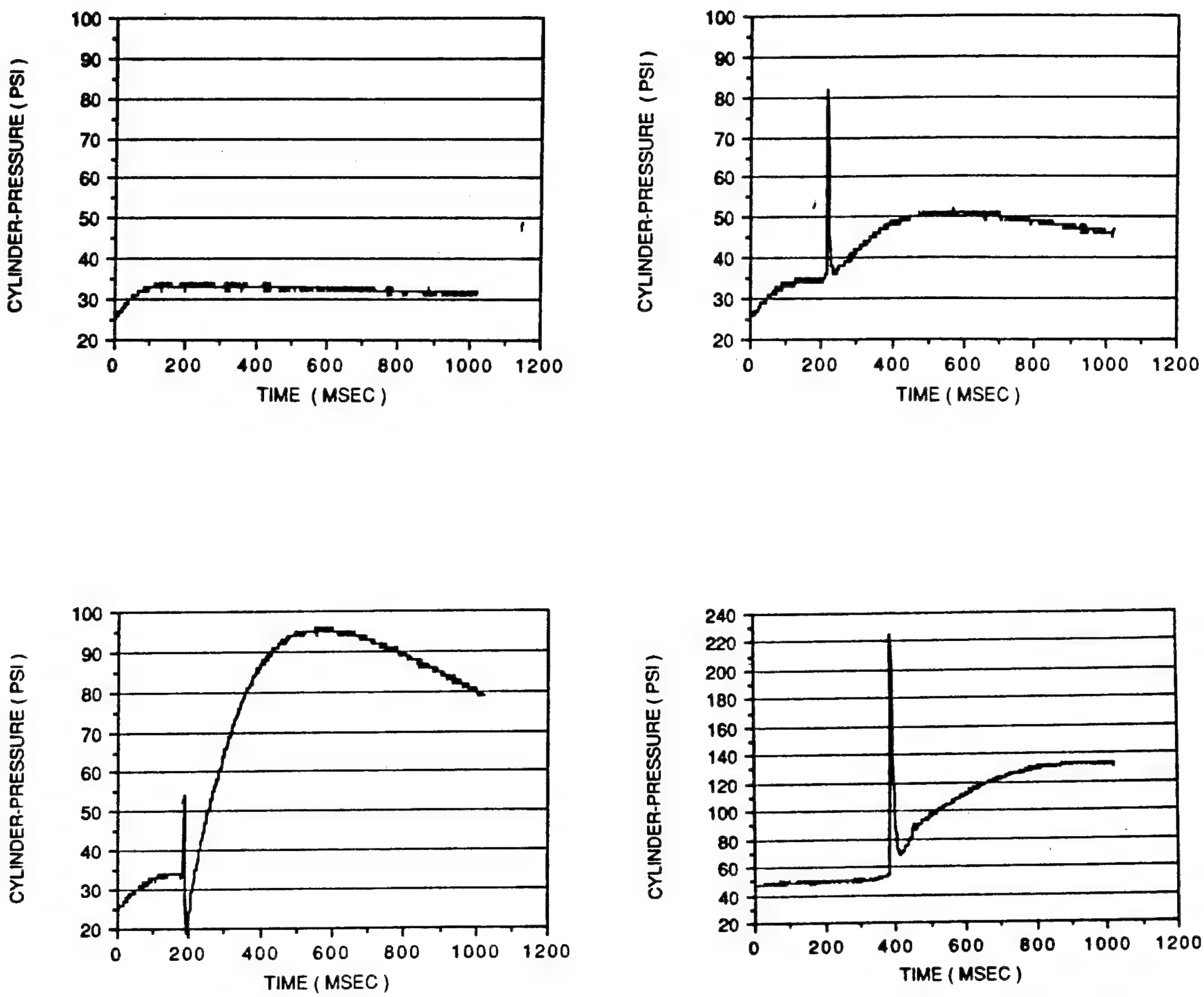

Figure 9–3: Pressure vs time for various mixtures of air and hydrogen, and for two settings of the pressure fuse. Top left: 20% hydrogen, 25 psi fuse, top right: 40% hydrogen, 25 psi fuse, bottom left: 60% hydrogen, 25 psi fuse, bottom right: 60% hydrogen, 50 psi fuse.

To determine the effect of the compression ratio on the performance of an internal combustion actuator, the pressure fuse trip point was increased from 0 to 50 psi holding all other variables constant. The pressure fuse trip point is measured relative to the cylinder pressure before the piston moves. At 0 psi, the compression ratio was 1:1, and there was a slightly audible pop but no significant pressure rise. At 50 psi the high pressure spikes were between 200 and 300 psi, and they sometimes ruptured hoses. No higher pressures were tried because we did not want to destroy the apparatus. Compare the data in figure 9–3 bottom left, which was produced with a 25 psi fuse, with that in figure 9–3 bottom right, which had a 50 psi fuse. The same amount of hydrogen and air (and thus the same potential energy content) was present in the cylinder in all cases. There is more "bang for the buck" at higher compression ratios. The charge could be injected into the cylinder at these high pressures negating the need for a compression stroke and releasing the same energy.

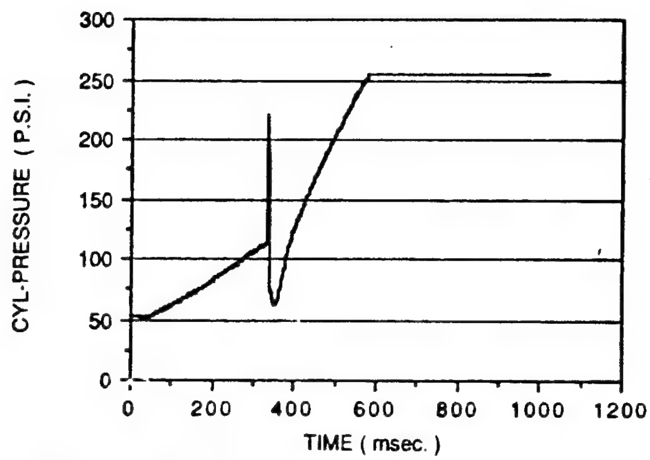

Figure 9–4: Pressure vs time for two stroke cycle.

Experiment 3: Combustion Cycle

Three types of combustion cycles were tested: 4 stroke, 2 stroke and 1 stroke. The four stroke cycle consisted of 4 sequential steps: 1) exhaust the cylinder 2) induct a fresh charge 3) compress the charge 4) explode the charge

Of all the cycles tried, this one was the easiest to implement. The shuttling of gasses from carburetor to cylinder and from cylinder to exhaust was straight forward with only a single state present in the cylinder at a time. The viability of the cycle did not seem to sensitively depend on small changes in parameters. Correspondingly, this cycle was chosen for most of the experimentation.

The two stroke cycle consisted of two steps: 1) exhaust water vapor and induct fresh charge 2) compress charge and explode

This cycle was difficult to implement because by its very definition exhaust gasses are exiting the cylinder at the same time a fresh charge is being drawn in. Timing is crucial. If the exhaust valve stays open too long all the new charge blows out with the old exhaust. If

it closes too quickly the old exhaust dampens and smothers the explosion of the new charge. This cycle seemed to be very dependent on small changes of parameters and was prone to misfiring and drowning in liquid residue of previous explosions. Although the actuator was eventually able to run consistently, a change in port design and location would greatly boost its reliability.

The one stroke cycle consisted of directly injecting a high pressure charge into the piston cylinder with the piston in the position of normal full compression and then exploding the charge. Although this cycle worked, producing reliable repetitive cycles was difficult because of the exhaust port placement.

Experiment 4: Frequency of Firing

The frequency of firing is the total time elapsed between subsequent explosions. It is a function of the response time of the solenoid valves used, the size of the internal passages and orifices, and the pressures in the carburetor and cylinder. It is also a function of the chosen combustion cycle. In practice, the maximum frequency of firing of this internal combustion actuator was one cycle every three seconds. This was caused by the time needed to recharge the carburetor chamber and the flow restrictions that limited the speed of the load simulating air cylinder. This firing speed is probably too slow for practical actuation. Replacing the carburetor with direct fuel injection into the piston chamber could reduce the cycle time measurably.

Experiment 5: Type of Load Applied

The type of load applied to the actuator piston is governed by the valving and regulation driving the load-simulating air cylinder. For most of the experiments this cylinder simply supplied a constant pressure to the actuator piston during an explosion. However, close observation revealed significant short term dynamic oscillations at the junction of the actuator piston and the rubber shock absorber that attaches it to the load cylinder. Other valving schemes were tried but the fundamental question of whether any of these faithfully represent a true load remains unanswered. Probably the best solution would be to replace the load air cylinder with an actual swinging mass leg.

9.2.2 Direct Fuel Injection

In order to address the problem of low firing frequency the carburetor on the test fixture was removed and replaced with a direct fuel injection system, shown in figure 9-5. This system consisted of two, two-way highspeed normally-closed solenoid valves and two manually adjustable flow regulators. In theory, for a 4-stroke cycle, the system should work in the following way:

1. Exhaust stroke—during the exhaust stroke as the piston descends both the exhaust

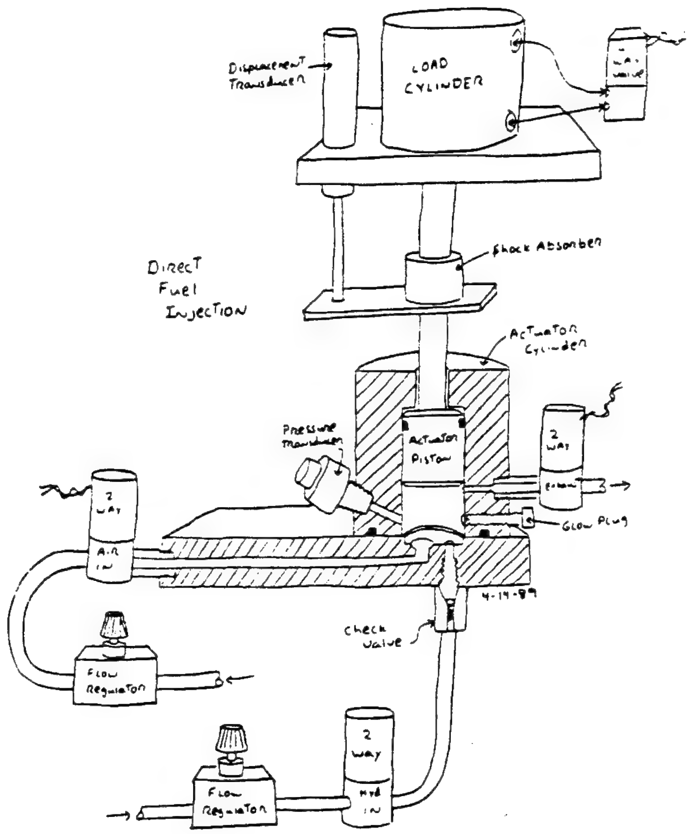

Figure 9–5: One sided internal combustion actuator with direct fuel injection.

and the air injection valve open forcing out the combustion byproducts and any water vapor that condensed on the cylinder walls.

2. Intake stroke—as the piston rises, both the air and hydrogen valves open allowing both gasses to rush into the chamber created as the piston moves. The fuel/air mix ratio achieved is governed by the pressure regulator and flow regulator setting for each of the component gasses.

3. Compression stroke—as the piston descends all valves close and pressure rises sharply in the cylinder.

4. Power stroke—at a given preset pressure the glowplug ignites the charge driving the piston up in the cylinder.

Results

The installation of a fuel injection system increased the firing frequency of the IC actuator

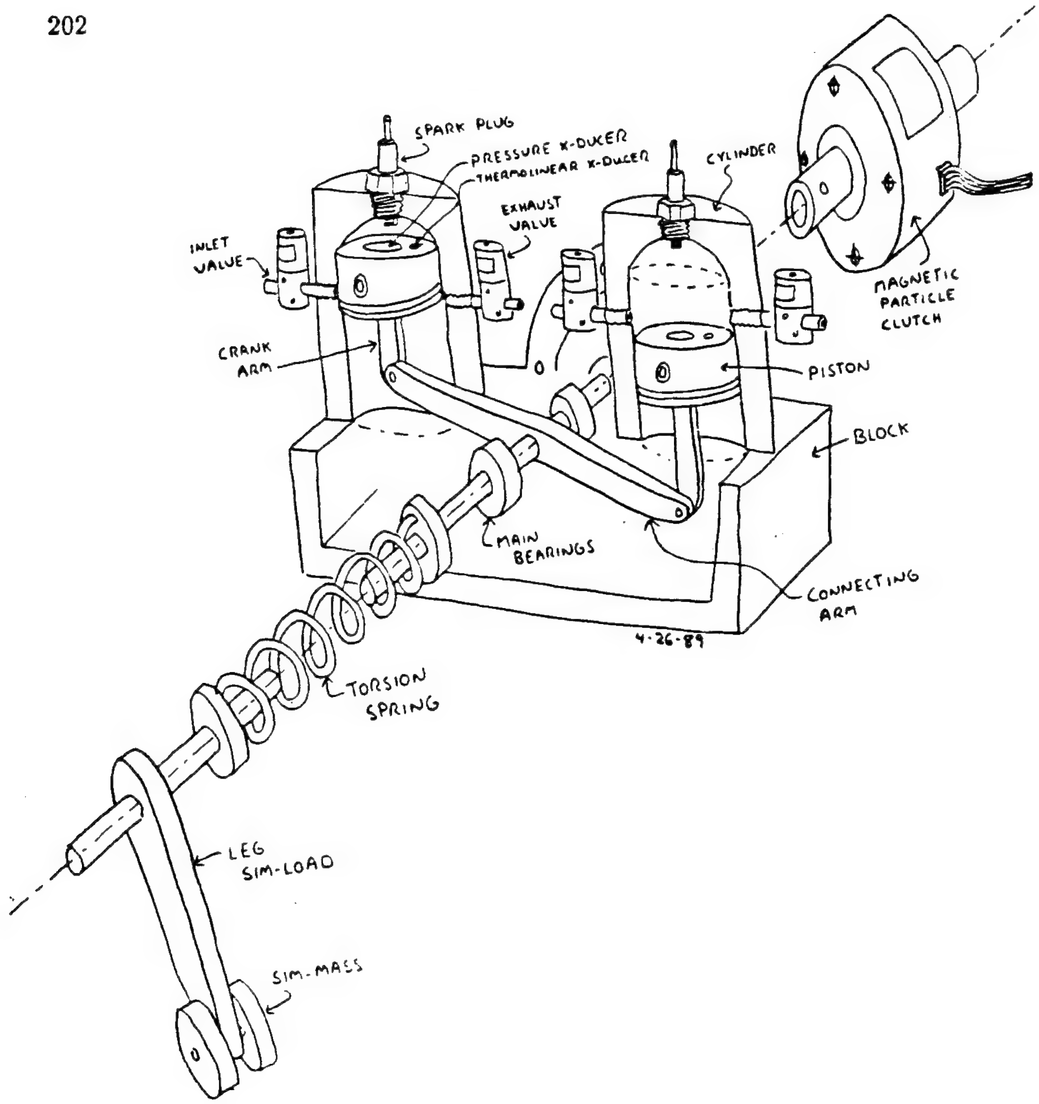

Figure 9–6: Concept for actuator driving leg oscillation.

which now seems to be limited only by the response time of the load simulating air cylinder. However, the addition of the four manually adjusted analog pressure and flow regulators considerably reduced the capability of precisely controlling the air/fuel mix ratio. It seems as though the pressure and flow settings for each gas (air and hydrogen) are tightly coupled to each other and to the firing frequency in as yet undetermined ways. Small changes in one variable (1/8 turn on a flow regulator) are enough to spell the difference between no explosion and large explosions. A 4-stroke cycle was started fairly quickly with a lot of knob tweaking (see figure 9–4), but the 2-stroke was not consistently firing. Further work needs to be done to improve the control of this system.

9.3 What Next?

We are considering the next step in the development of the internal combustion actuator. One plan would be to build a system coupled to a leg-like mechanism. One concept is shown in figure 9–6. The device consists of three main parts:

1. A swinging mass leg driven by a torsion spring. This leg could have variable length and mass and the corresponding drive spring could be easily replaceable so that a reasonable spectrum of leg loads could be simulated. As an actual swung mass load, this system would accurately simulate real leg loads where the previous high friction air piston failed.

2. A double acting internal combustion actuator.

3. High speed magnetic particle clutch that will allow the actuator to be instantly frozen in a desired location and held there as long as necessary.

Some of the design features and goals of the system are

- Two pistons for 2-way motion.
- High compression cylinder heads, with no fittings, hoses, or valves in the high compression zone.
- High flow opposing port design to combat fouling and promote water vapor purging.
- Better piston stroke to bore ratio to promote heat retention for water vaporization.
- Integral piston pressure and temperature sensors for accurate chamber state measurements.
- Spark ignition system for rapid controllable firing.
- A controllable fuel injection system using massflow sensors and a closed loop control.
- A precision potentiometer to measure the displacement of the piston in the cylinder.

9.4 Summary and Conclusions

Two internal combustion actuators were designed, fabricated and tested in order to determine the feasibility of using combustion as a primary driver for locomotion. In the process of experimentation, the actuators were modified considerably to improve their performance. The results of this experimentation are listed as follows:

1. Explosions in a cylinder with hydrogen as a fuel are fairly easy to produce.

2. The force produced by the explosion is directly linked to the fuel/air mix ratio in a repeatable, predictable way.

3. For a given mass of fuel/air charge, the higher its compression before ignition, the more energy is released in its explosion.

4. 4-stroke combustion cycles are inherently easier to implement than 2 or fewer strokes. However, the lower strokes produce more power per piston cycle.

5. Port location and configuration is crucial to a workable 2-stroke design.

6. The load the actuator sees is a major contributor to its dynamic explosion response.

In short it appears possible to produce controlled explosions of a given force at potentially high frequencies in an internal combustion actuator with existing technology.

Chapter 10

Zero Gravity Running

10.1 Introduction

Legged locomotion is usually associated with an environment having a gravitational field. Normally it is not possible to run without gravity, because the upward motion that initiates the flight phase can not be reversed once contact with the ground is lost. One solution would be to use sticky or prehensile feet. In this case the flight phase would be replaced by a "tension phase" during which the vertical motion of the body would be reversed as tension in the legs pulled the body downward toward the support surface.

Another solution, first suggested by Robert Cannon (1985), is to run between two floors. In this case, the supporting forces generated in the collision with one floor reverses the vertical velocity of the previous collision with the other floor. Such a configuration for running might exist in a space station where the walls, floors, and ceilings could act as rebounding surfaces.

We studied a machine designed for running in zero-gravity. The machine has one leg and two feet, and it is restricted to move in plane. The machine uses a control strategy developed for control of hopping machines. Algorithms that control running in zero-g can be similar to those used in one-g, to the extent that behavior during ground collision in normal running is dominated by the need to reverse the vertical velocity of the body, rather than oppose the acceleration of gravity. This paper reports computer and laboratory simulations that test this idea.

10.2 Similarities Between Biped Running and Zero Gravity Running

The basic approach to controlling the running in zero gravity is to use algorithms originally developed for a one-gravity biped. The two tasks are similar in three respects. First, the

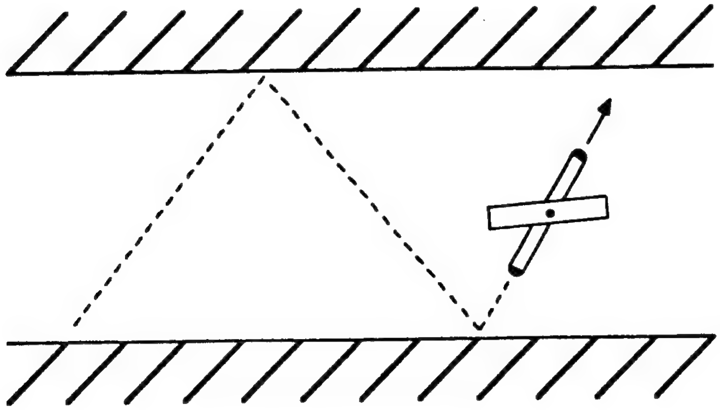

Figure 10-1: The zero gravity running machine rebounds from two parallel floors as it travels down the corridor.

phases of the running cycle are identical. Four consecutive phases complete one cycle (see diagram):

- stance phase on leg one
- first flight phase
- stance phase on leg two
- second stance phase
- repeat cycle.

Second, the flight phases of the two systems are similar. For the system running in one-gravity environment, the velocity just after take-off is equivalent to the velocity just prior to touchdown because the flight is ballistic. Likewise, in a zero-gravity environment, the corresponding velocities are also equivalent because the velocity is constant throughout the flight phase.

The stance phases are also similar. Behavior during the stance phase for hopping with or without gravity is similar to the behavior of a mass-spring oscillator. The velocity at the beginning of the stance phase is equivalent to the velocity at the end of the stance phase. Also, for both systems, there is a change in direction during the stance phase.

10.3 Control of Zero Gravity Running

The similarities between the two systems allowed us to use the same control algorithm. This control algorithm is responsible for maintaining balance and for alternating active and idle legs. It is robust enough such that the absence of gravity does not introduce any additional control problems.

The control algorithm uses a state machine to synchronize control actions with the events in the running cycle. After each unloading state, the idle leg becomes active and the

active leg becomes idle. Because the zero-gravity running machine has only one leg, the idle leg cannot be moved independently and thus it automatically opposes the motion of the active leg. This motion, like the mirroring motion of the biped, positions the idle leg near the correct leg position for the next stance cycle. Also the idle leg does not have to retract since there is no danger of striking the ground as it swings forward for the next touchdown.

10.4 Computer Simulation of Zero-Gravity Running

A model of the zero-gravity running machine was simulated. The dynamic model consisted of a body, an upper leg, a massless lower leg, and two massless feet. A drawing of the model is shown in figure 10–2. The massless lower leg slides into the upper leg like a pogo stick. The feet attached to the ends of the lower leg were used to model ground interaction. The machine has two actuators, one on the hip and one on the leg. The hip actuator was modeled as a torque source that acts between the body and the leg. The leg actuator was modeled as a force source that acts between the upper and lower parts of the leg. The output torque and force were calculated by the control program.

A computer program performed control calculations and numerical integration of the dynamic equations derived from the model. The control calculations monitored sensor data from the dynamic model to synchronize the controller state machine with the model, and to calculate the actuator torque and force. A variable step Runga-Kutta algorithm integrated the dynamic equations of the model with initial conditions and control calculations from the previous cycle. Data from the simulation was recorded for future playback as plots or animated cartoons.

The bouncing motion of the machine between two walls can be seen in the path of the body in figure 10–1.

10.5 Zero Gravity Running Machine

The successful simulation of zero-gravity running encouraged us to design and build a physical machine demonstration of zero-gravity running. The machine has a single leg with a telescoping airspring on each end. The leg is connected to the body by a pivot joint that forms the hip. A pneumatic cylinder actuator at the hip positions the leg. The air springs on the leg make the leg springy and also serve as thrust actuators. A two dimensional zero gravity environment was created by floating the machine on a smooth table with air bearings. The table was placed in a hallway where the walls acted as rebound surfaces for the legs.

An onboard computer monitors the sensors, performs the control calculations, and sends commands to the actuators. As in the simulation, the control program synchronizes the state machine with the actuators via sensor inputs. In each state, the appropriate servo calculation is performed and results are output to the actuator.

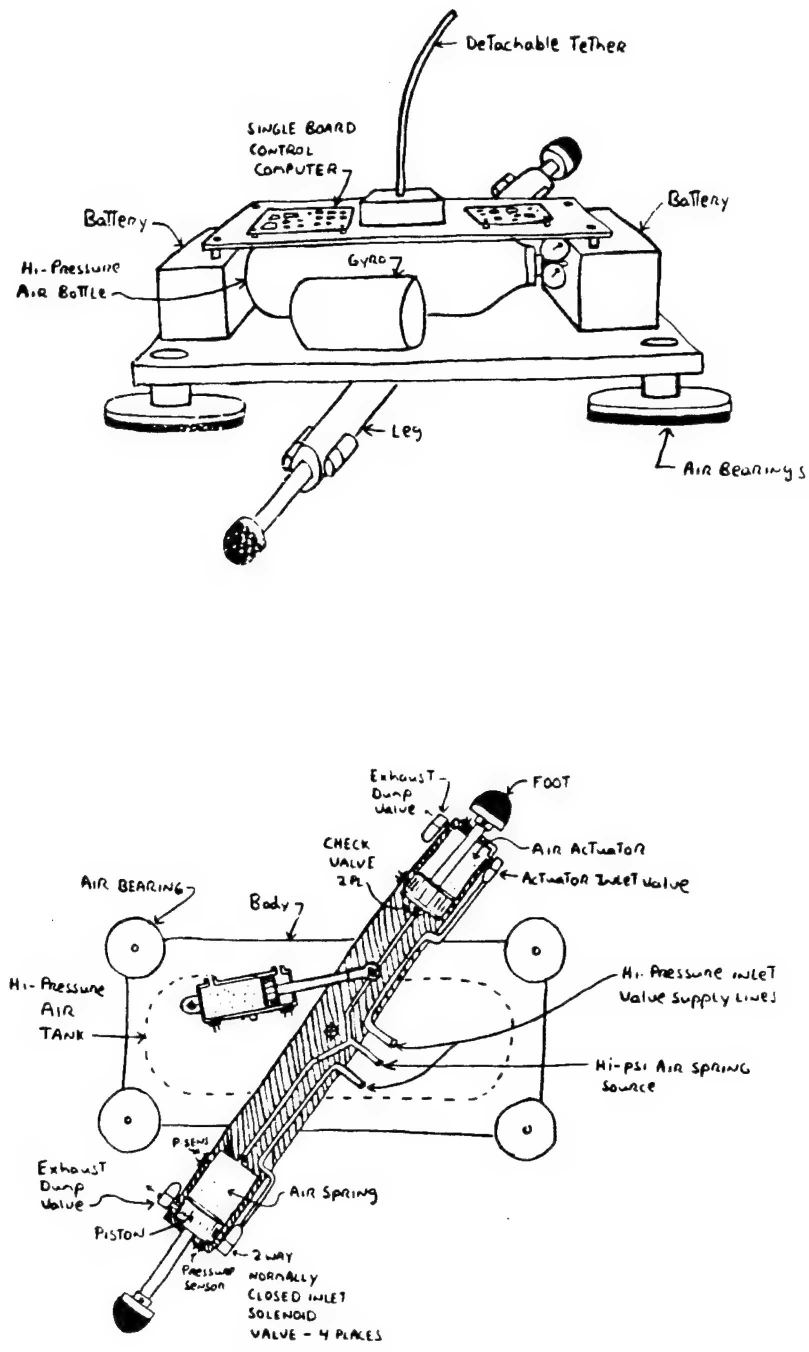

Figure 10–2: Diagram of planar zero-gravity running machine.

10.6 Summary

Running can take place in an environment without gravity, if there are two support surfaces to bounce between. The control algorithm developed for a bipedal running machine are adequate to control running in zero gravity. This is possible because the running cycles of both machines are the same, and the absence of gravity has a small effect on the dynamics of the machine.

10.7 Appendix: Detailed Description of the Zero-Gravity Running Machine

A frame made out of half inch birch plywood separated by 6 inch aluminum dowels holds electronics and sensors on the top level, air tank, batteries, and hip actuator on the middle level, and air bearings, leg and thrust actuators on the bottom level.

The hip, a steel dowel supported by two bearing pillow blocks attached to the plywood frame, supports the leg. A rotary potentiometer attached to the top of the hip dowel measures hip angle. The hip actuator is a single ended cylinder attached to the dowel through a lever arm. The cylinder is controlled by a servo valve which creates a pressure difference across the cylinder proportional to input current.

The leg has two double-ended pneumatic cylinders which act as both air springs and thrust actuators during the stance phase. Two rubber feet are attached to the ends of the cylinder rods. Switches inside the feet close when ground contact is made. Pneumatic valves control the thrusting of the leg.

Six sensors were used: two foot switches to sense ground contact, two linear potentiometers to measure leg length, a rotary potentiometer to measures hip angle, and a free gyroscope to measure body angle relative to the orientation of the support surface. The control computer calculates derivatives to find the hip rate, body pitch rate, and leg length rate.

Three air bearings attached to the bottom of the frame float the machine on the floor and allow it to slide with very little friction. The air bearings are 6 inch diameter discs cut from 1/2 inch aluminum plate. Flow control valves regulate the flow of air through 1/4 inch holes drilled in the center of the plates.

The electronics interface and control computer for the zero-gravity hopper are all located on the machine. The electronics interface has an analog and digital component. The a 16 channel analog to digital converter and a four channel digital to analog converter. All control calculations for the zero-gravity hopper are done on board with a general purpose digital signal processor computer. The development interface is a dual port ram connected to a VAX 785 computer.

Power for the zero-gravity running machine is both electric and pneumatic. The machine is designed to operate without a tether for control or power. The electrical power system was designed to be autonomous and an attempt was made to make the pneumatic system autonomous. The electrical power system consists of a bank of thirty 4.5 amp-hour nickel cadmium batteries connected in series, which provide 36V-30V during discharge. DC to DC converters provide 5V@5A and +/- 15V@750mA for the computer and analog electronics. The solenoid valves use an unregulated centertap 24V from the batteries. A 30V DC to 115VAC 400Hz converter powers the gyroscope. Fully charged batteries are capable of powering the system for approximately half an hour, if the gyroscope is not caged frequently.

Due to the properties of the servo valve and the air bearings, the pneumatic system

is far from autonomous. A 2.5 gallon high pressure tank was placed on board to provide autonomous pressure, but it was found that the charge in the cylinder was enough to provide lift for only 40 seconds. Much lower flow rates could be attained if both the air bearings and the floor surfaces were smoother. Another source of air loss is the pneumatic servo valve for the hip actuator, which uses a high flow, even when the actuator is stationary. The performance of the system could also be improved if the total weight were reduced. The batteries and high pressure steel tank with regulators add about twenty five pounds bringing the total system weight to sixty pounds.

To run the machine and record data, the onboard computer was connected to a VAX computer. The computer provided parameters like joystick input, servo gains, and start and stop signals. The VAX recorded data from all the sensors.

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